



COMPARTMENTAL ANALYSIS OF DRUG DISTRIBUTION

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DRUG DISTRIBUTION

The post-absorptive transfer of drug from one location in the body to another.

- **Compartmental Models**
(ordinary differential equations)
- **Distributed Models**
(partial differential equations)

Pharmacokinetic Models Using Ordinary Differential Equations*

MODEL	NUMBER OF COMPARTMENTS	MATHEMATICAL CHARACTERISTICS
NONCOMPARTMENTAL	0	CURVE FITTING TO DATA
COMPARTMENTAL	1 – 3	MODEL PARAMETERS FIT TO DATA
“PHYSIOLOGICAL”	4 - 20	MODEL PARAMETERS FIXED A <i>PRIORI</i>

* From Atkinson AJ Jr, et al. Trends Pharmacol Sci 1991;12:96-101.

Mathematical vs. Physical Models*

MATHEMATICAL MODEL:

Functions or differential equations are employed without regard to the physical characteristics of the system.

PHYSICAL MODEL:

Implies certain mechanisms or entities that have physiological, biochemical or physical significance.

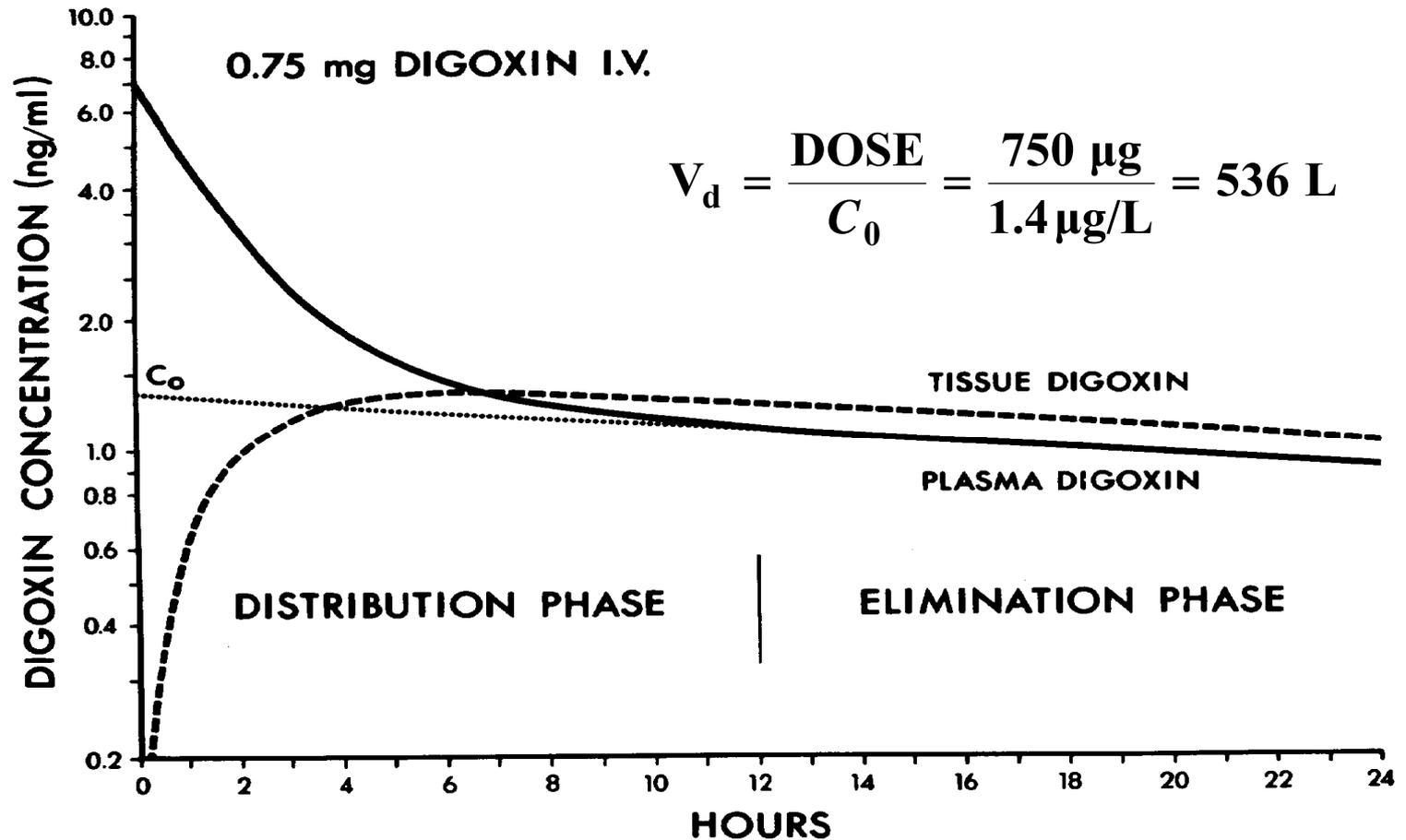
* Berman M: The formulation and testing of models.
Ann NY Acad Sci 1963;108:182-94

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Goals of Drug Distribution Lecture

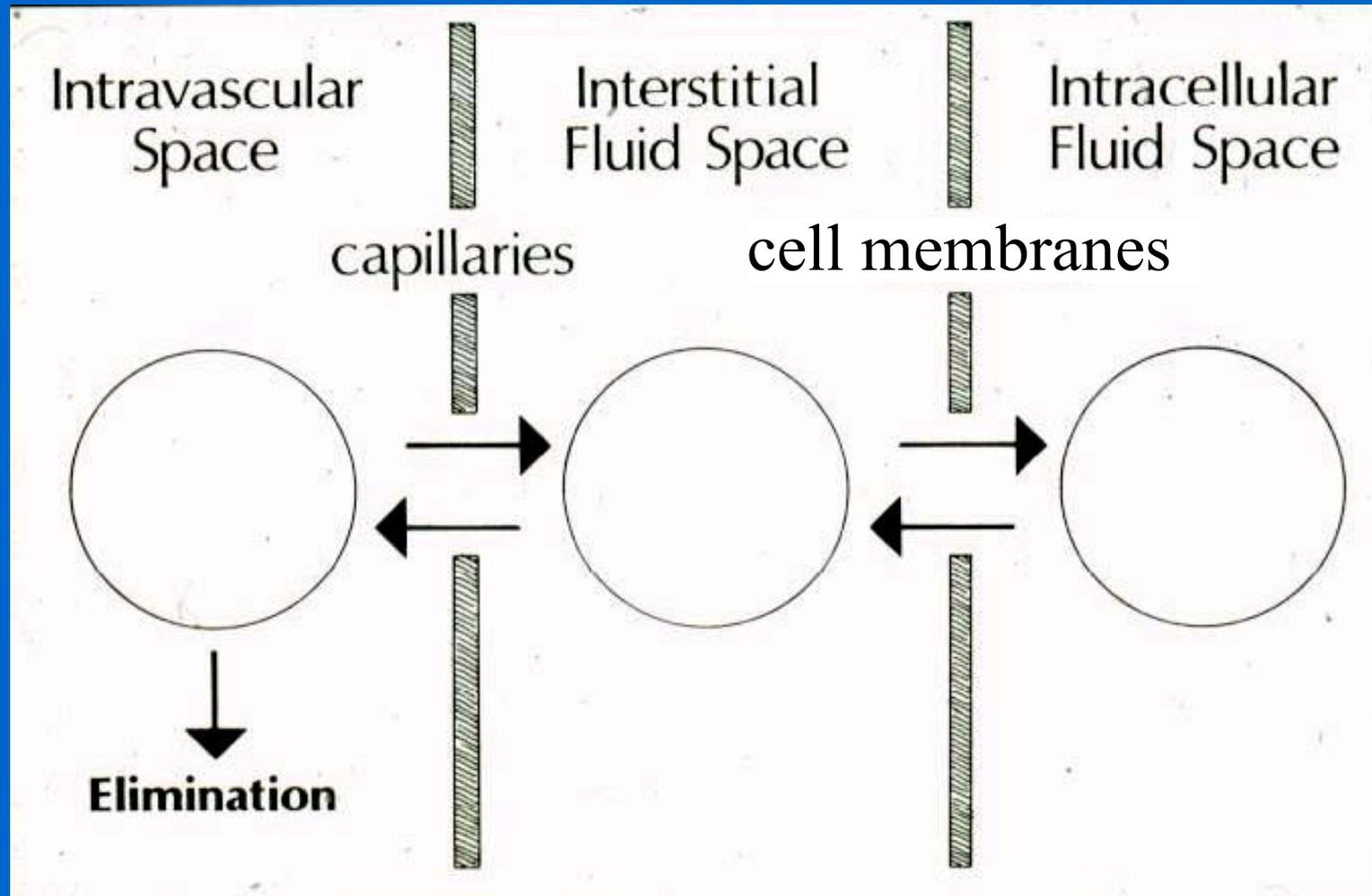
- **Significance** of Drug Distribution Volumes
- **Physiological Basis** of Multi-Compartment Pharmacokinetic Models
- **Clinical Implications** of Drug Distribution Kinetics

DIGOXIN DISTRIBUTION VOLUME



Body Fluid Spaces

Catenary 3-Compartment Model



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Volume of Distribution and Physiological Fluid Spaces

Intravascular Space:

None

Extracellular Fluid Space:

Inulin

Proteins and other Macromolecules

Neuromuscular Blocking Drugs (N^+)

Aminoglycoside Antibiotics (initially)

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Volume of Distribution and Physiological Fluid Spaces

Total Body Water

Urea

Ethyl alcohol

Antipyrine (some protein binding)

Caffeine

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Factors Affecting Volume of Distribution Estimates

Binding to Plasma Proteins

Thyroxine

Theophylline

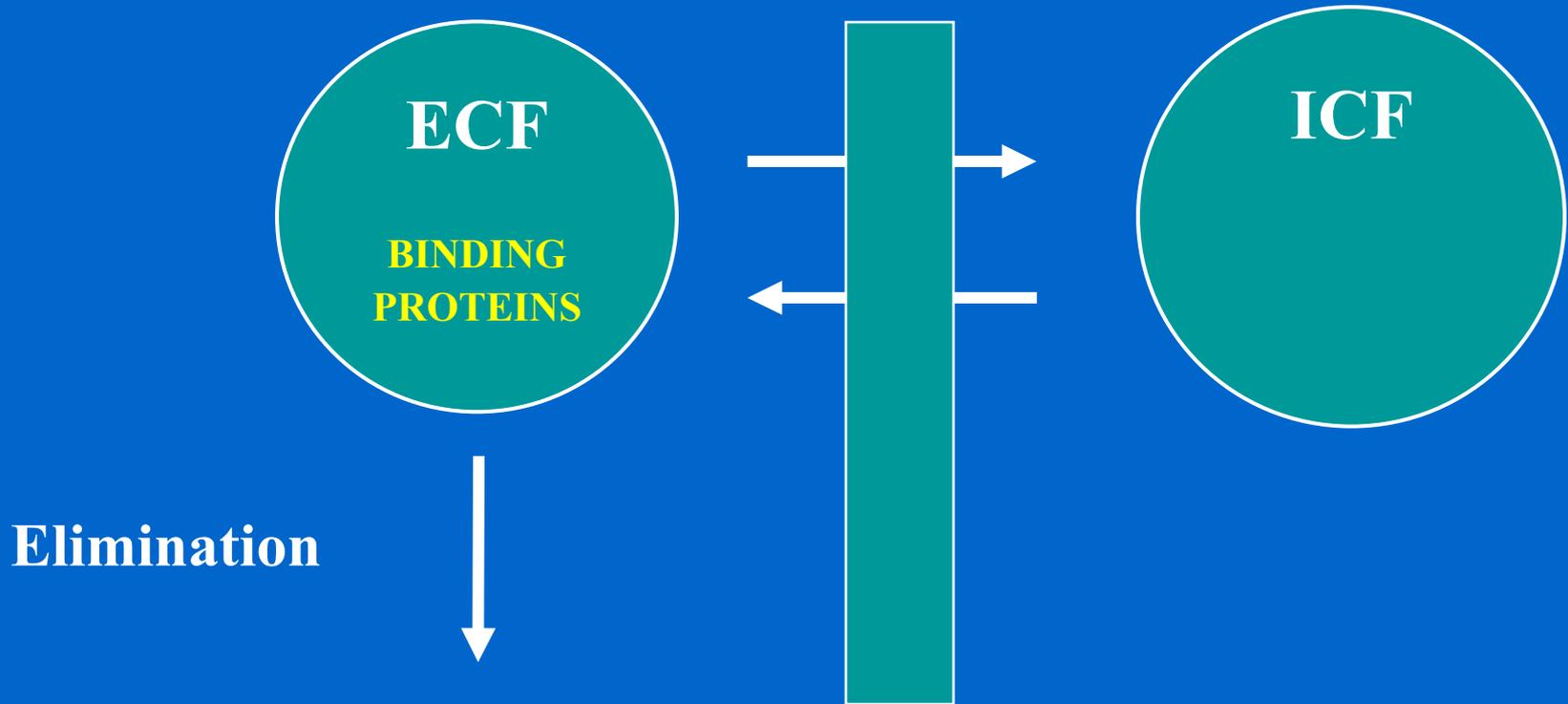
Tissue Binding (partitioning)

Lipophilic Compounds

Digoxin (Na^+ - K^+ ATPase)

Effect of Plasma Protein Binding on Drug Distribution

Cell Membranes



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Effect of Plasma Protein Binding on Apparent Volume of Distribution*

$$V_d = ECF + f_u (TBW - ECF)$$

f_u is the “free fraction”, the fraction of drug in plasma that is not bound to plasma proteins.

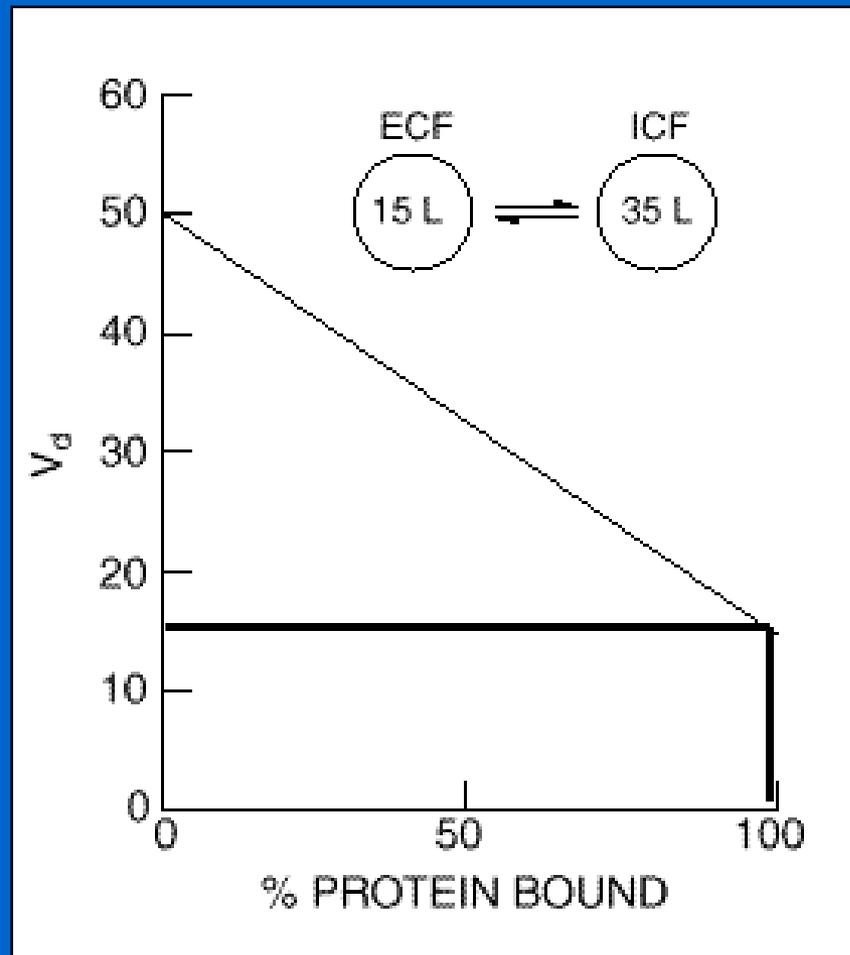
* Atkinson AJ Jr, et al. Trends Pharmacol Sci 1991;12:96-101.

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Impact of Protein Binding on Thyroxine Distribution Volume*

$$f_u = 0.03\%$$

$$V_d = V_{\text{ECF}}$$

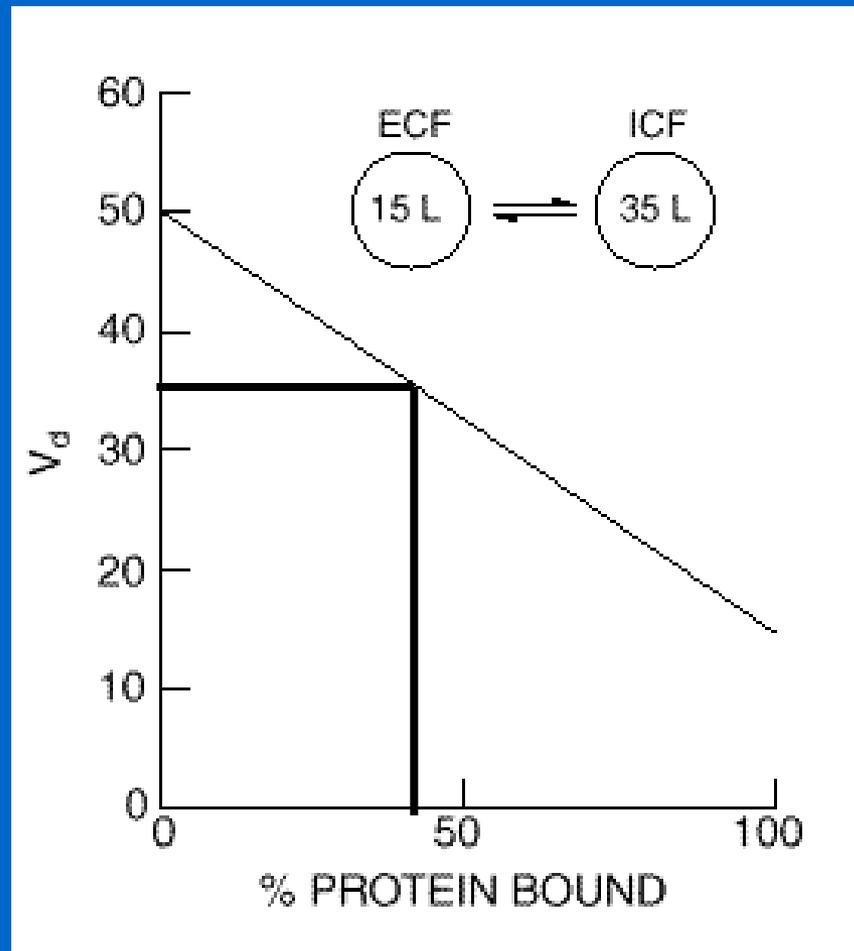


* From Larsen PR, Atkinson AJ Jr, et al. J Clin Invest 1970;49:1266-79.

Impact of Protein Binding on Theophylline Distribution Volume*

$$f_u = 60\%$$

$$V_d = V_{ECF} + f_u V_{ICF}$$



* From Atkinson AJ Jr, et al. Trends Pharmacol Sci 1991;12:96-101.

Basis for Increased **Theophylline** Volume of Distribution in Pregnancy*

	f_U (%)	FLUID SPACE ESTIMATES (L)		TOTAL V_d (L)	
		ECF	TBW	EST.	MEAS.
PREGNANT					
24-26 WEEKS	88.9	13	34	32	30
36-38 WEEKS	87.0	21	40	38	37
POSTPARTUM					
6-8 WEEKS	77.4	12	33	28	28
>6 MONTHS	71.9	12	33	27	31

* From Frederiksen MC, et al. Clin Pharmacol Ther 1986;40;321-8.

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Effect of Plasma Protein and **Tissue Binding** on the Volume of Distribution of Most Drugs*

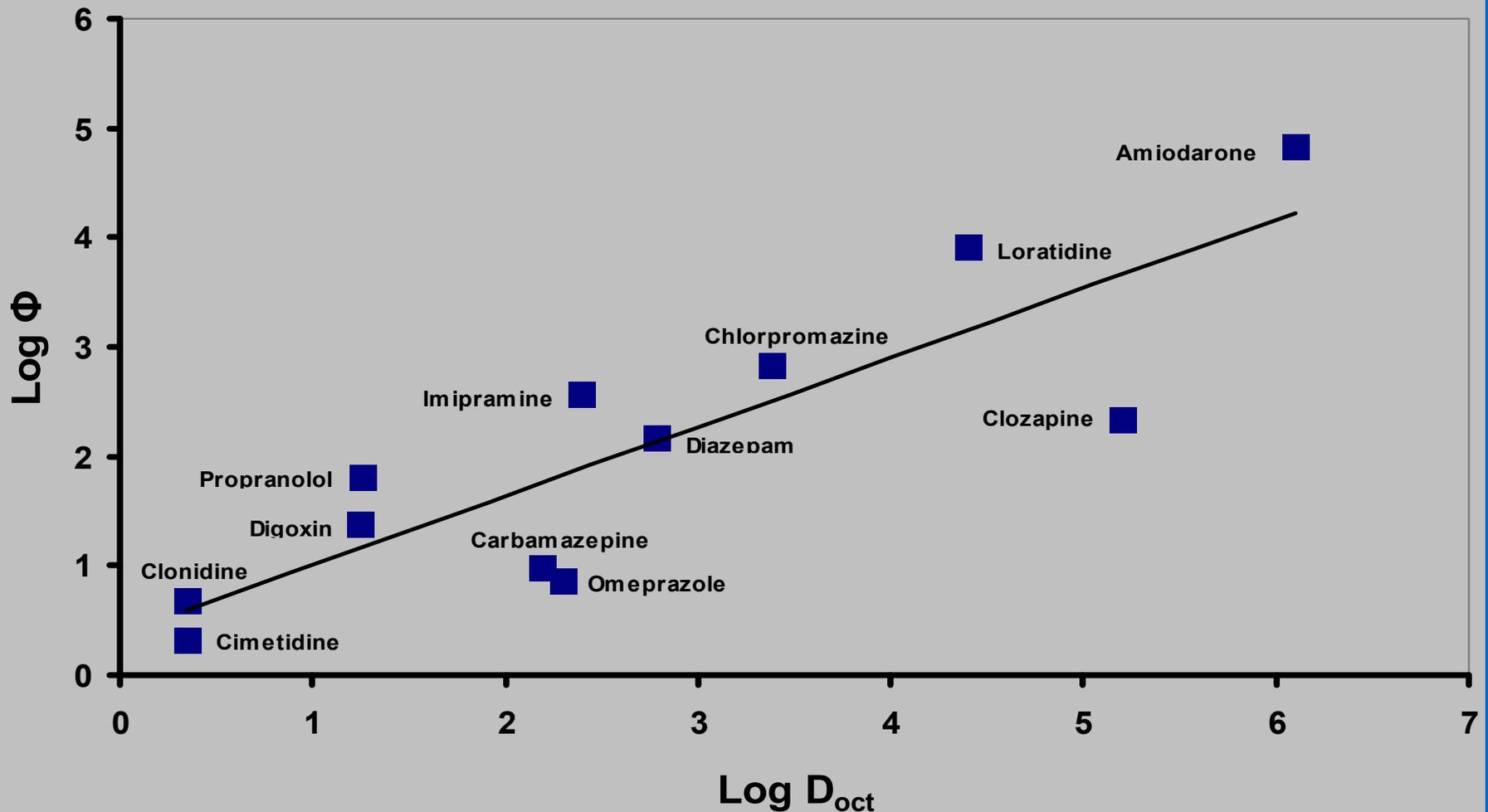
$$V_d = ECF + \Phi f_u (TBW - ECF)$$

Φ is the ratio of tissue/plasma drug concentration.

* Atkinson AJ Jr, et al. Trends Pharmacol Sci 1991;12:96-101.

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LIPID SOLUBILITY (D_{oct}) and Φ



Apparent Volume of Distribution for Digoxin

$$V_d = ECF + \Phi f_u (TBW - ECF)$$

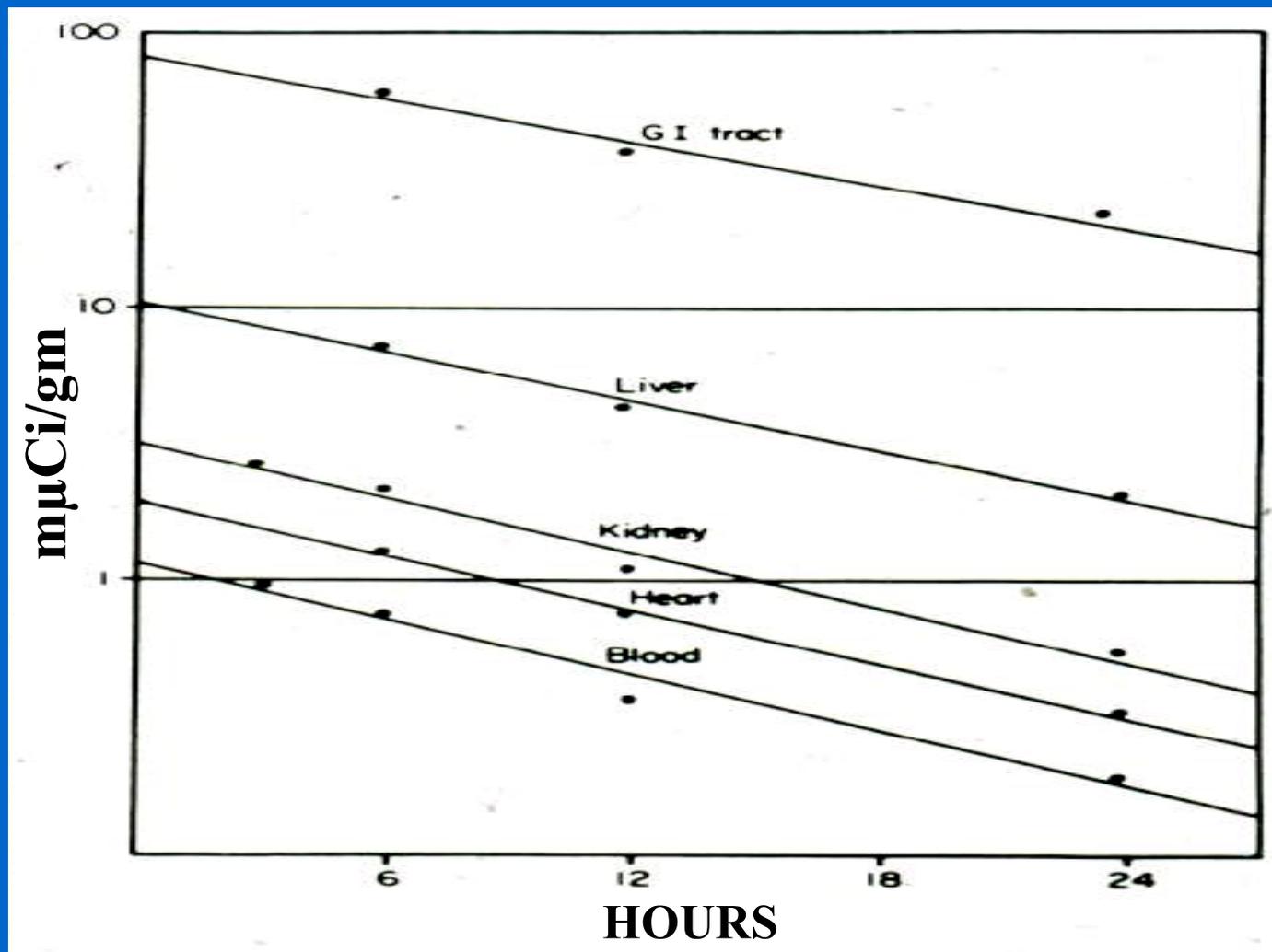
$$ECF = 11.2L, TBW = 45.5L, f_u = 0.75, \Phi = 20.4$$

$$V_d = 11.2 + (20.4)(0.75)(34.3) L$$

$$V_d = 536 L$$

Φ includes binding to $Na^+ - K^+$ ATPase.

Tissue vs. Plasma **Digoxin** Levels

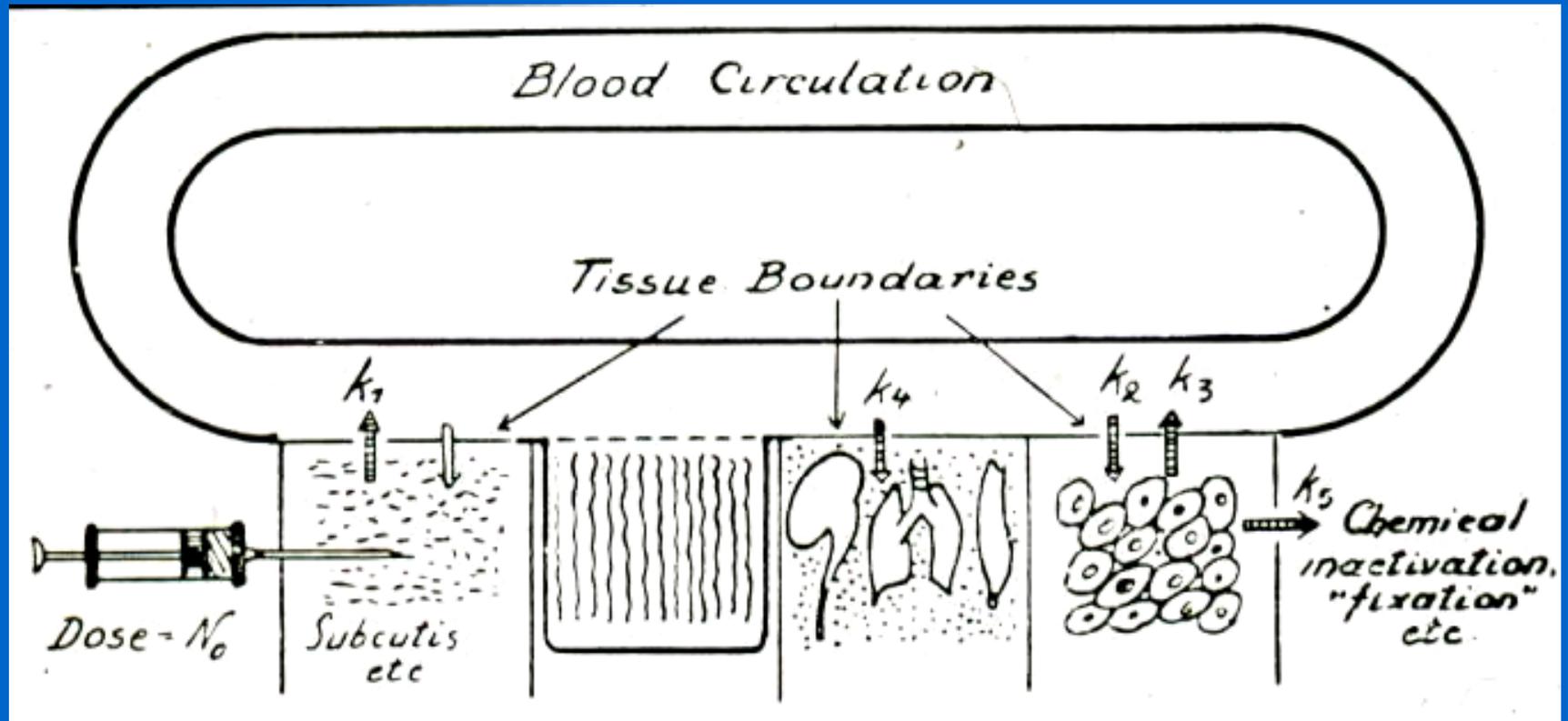


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GOALS OF DRUG DISTRIBUTION LECTURE

- **Significance of drug distribution volumes**
- **Physiologic basis of multi-compartment pharmacokinetic models**
- **Clinical implications of drug distribution kinetics**

First Multicompartmental Analysis of Drug Distribution*



* From Teorell T. Arch Intern Pharmacodyn 1937;57:205-25.

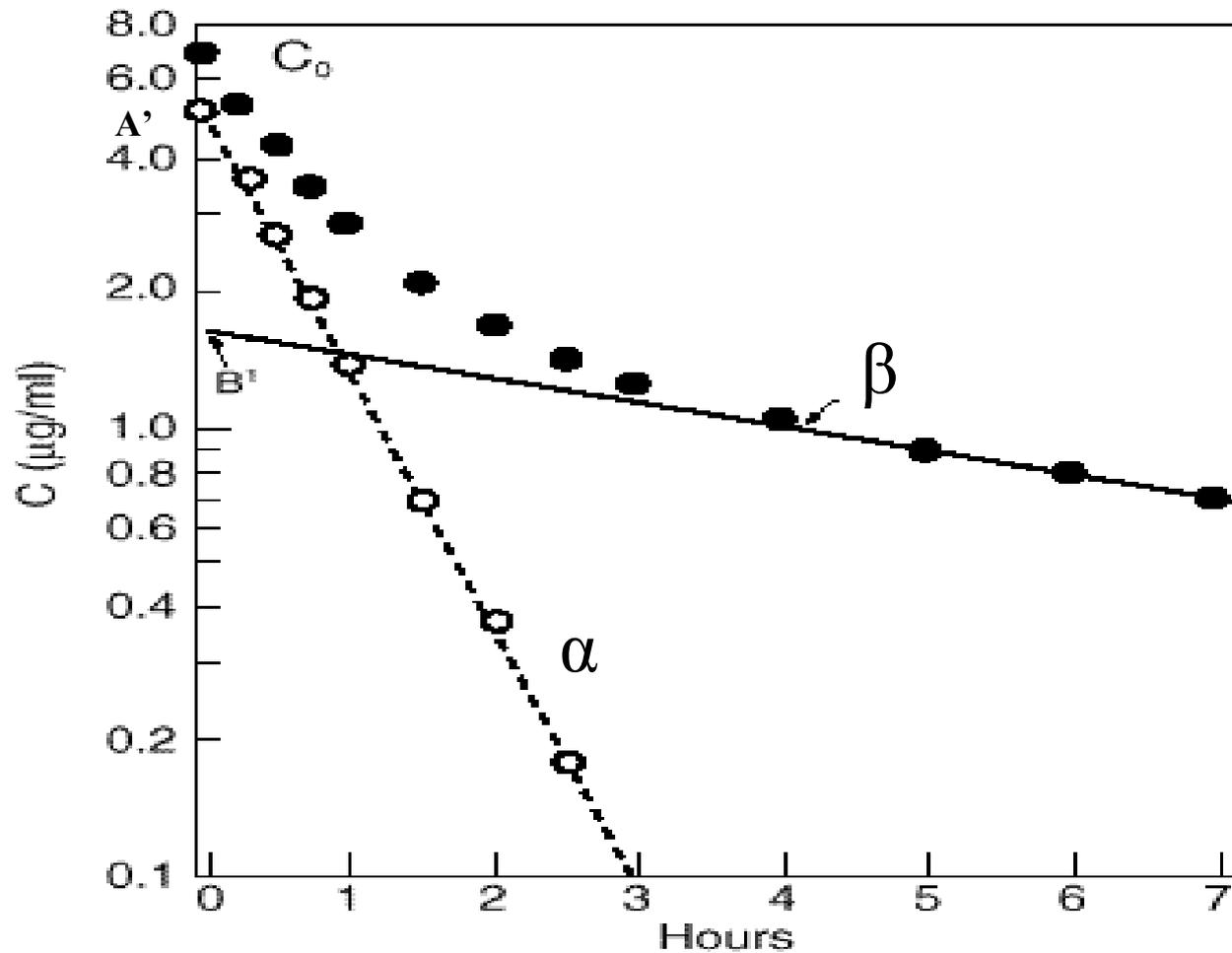
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Analysis of Experimental Data

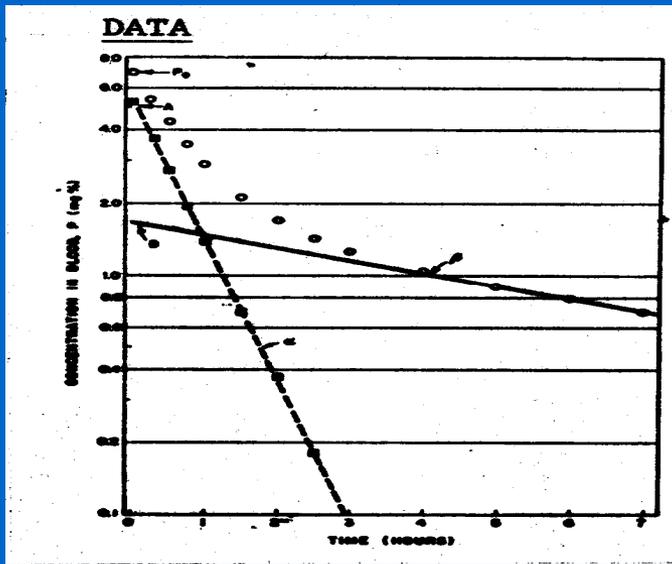
How many compartments?

*Number of exponential phases
in plasma level vs. time curve
determines the number of
compartments.*

TECHNIQUE OF *CURVE PEELING*

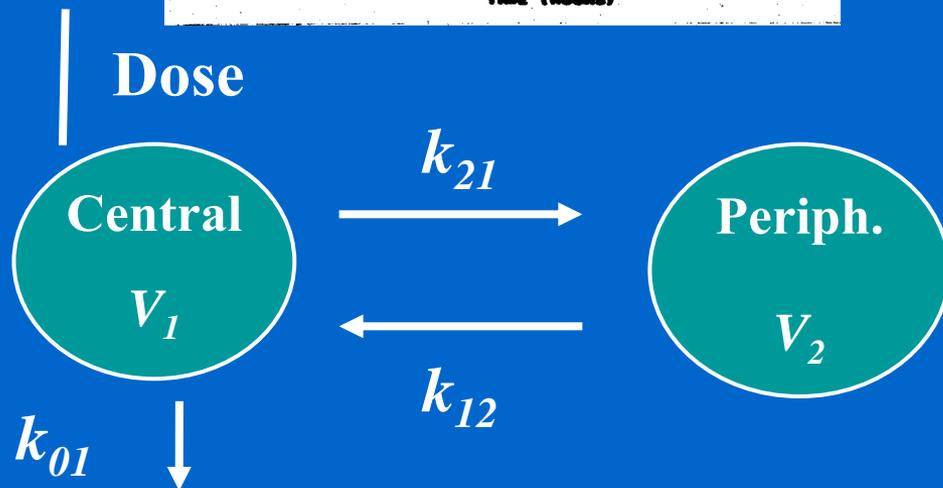


COMPARTMENTAL ANALYSIS



Data Equation:

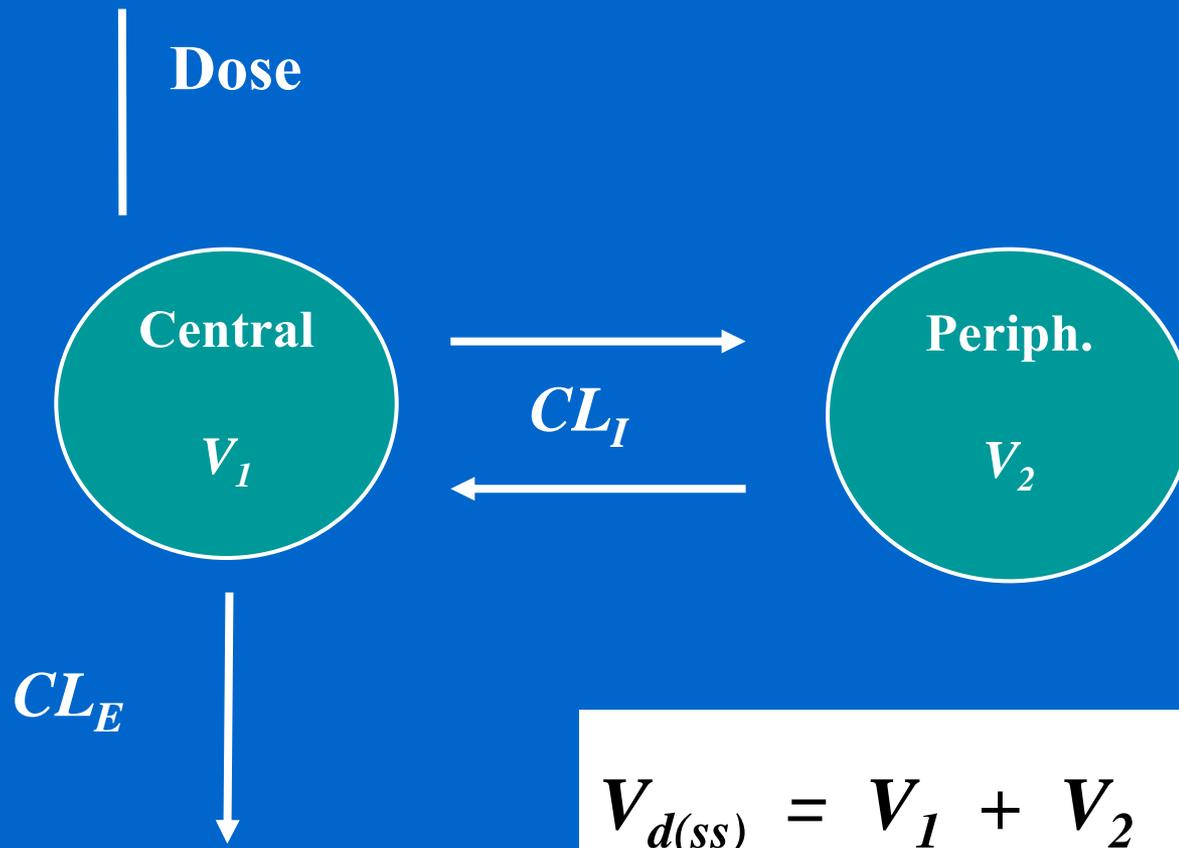
$$C = A'e^{-\alpha t} + B'e^{-\beta t}$$



Model Equation:

$$dX_1/dt = -(k_{01} + k_{21})X_1 + k_{12}X_2$$

TWO-COMPARTMENT MODEL



$$V_{d(ss)} = V_1 + V_2$$

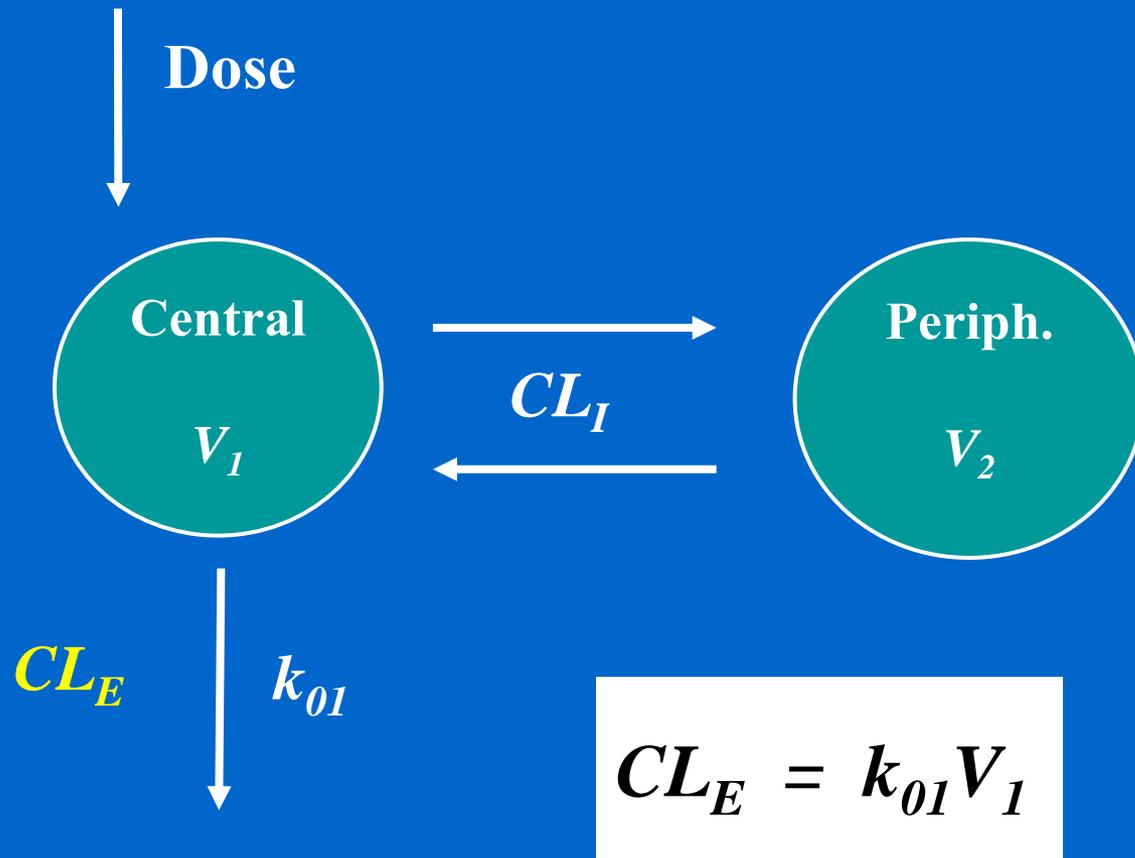
3 DISTRIBUTION VOLUMES

$$V_{d \text{ (extrap.)}} = \text{DOSE} / C_0$$

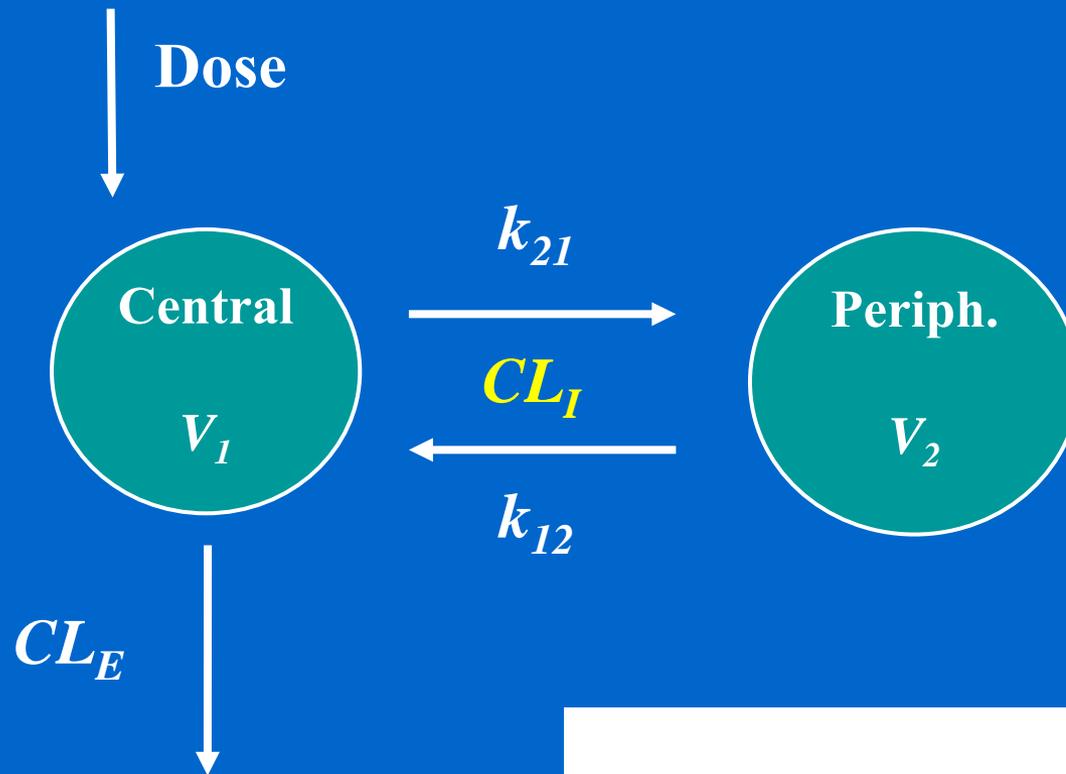
$$V_{d \text{ (area)}} = \frac{t_{1/2} \cdot CL_E}{0.693}$$

$$V_{d \text{ (ss)}} = V_1 + V_2 + \dots + V_n$$

TWO-COMPARTMENT MODEL



TWO-COMPARTMENT MODEL



$$CL_I = k_{21} V_1 = k_{12} V_2$$

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INTERCOMPARTMENTAL CLEARANCE*

**Volume-Independent Parameter
Characterizing the Rate of Drug Transfer
Between Compartments of a Kinetic
Model**

* From Saperstein et al. *Am J Physiol* 1955;181:330-6.

Is Central Compartment Intravascular Space?

- Usually **not** identified as such **unless** drug is given **rapidly IV**.
- **NEED TO CONSIDER:**
 - If distribution is **limited to ECF**, compare the central compartment volume with **plasma** volume.
 - If distribution volume **exceeds ECF** compare central compartment with **blood** volume.*

*(account for RBC/Plasma partition if [plasma] measured)

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Analysis of **Procainamide** and **NAPA** Central Compartment Volumes*

DRUG	V _c (L)	RBC/P	INTRAVASCULAR SPACE (L)	
			PREDICTED	OBSERVED
PA	6.7	1.52	5.6	5.5
NAPA	7.5	1.62	5.6	6.0

* From Stec GP, Atkinson AJ Jr. J Pharmacokinet Biopharm 1981;9:167-80.

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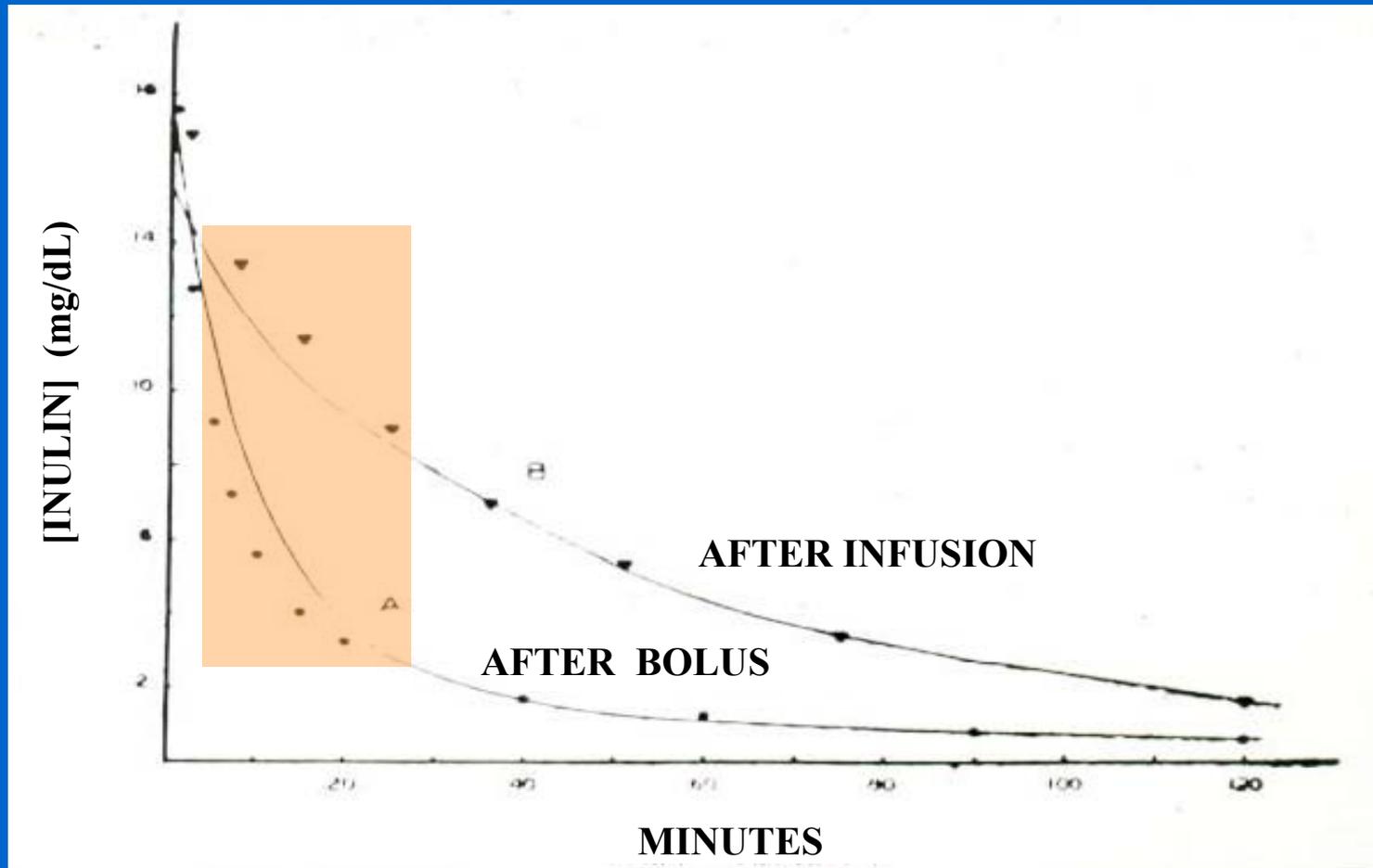
If Central Compartment Volume is Based on Plasma Concentration Measurements

$$V_{C(\text{corr.})} = V_{C(\text{meas.})} / [(1 - \text{Hct}) + \text{Hct}(\text{RBC/P})]$$

RBC/P = red cell/plasma partition ratio

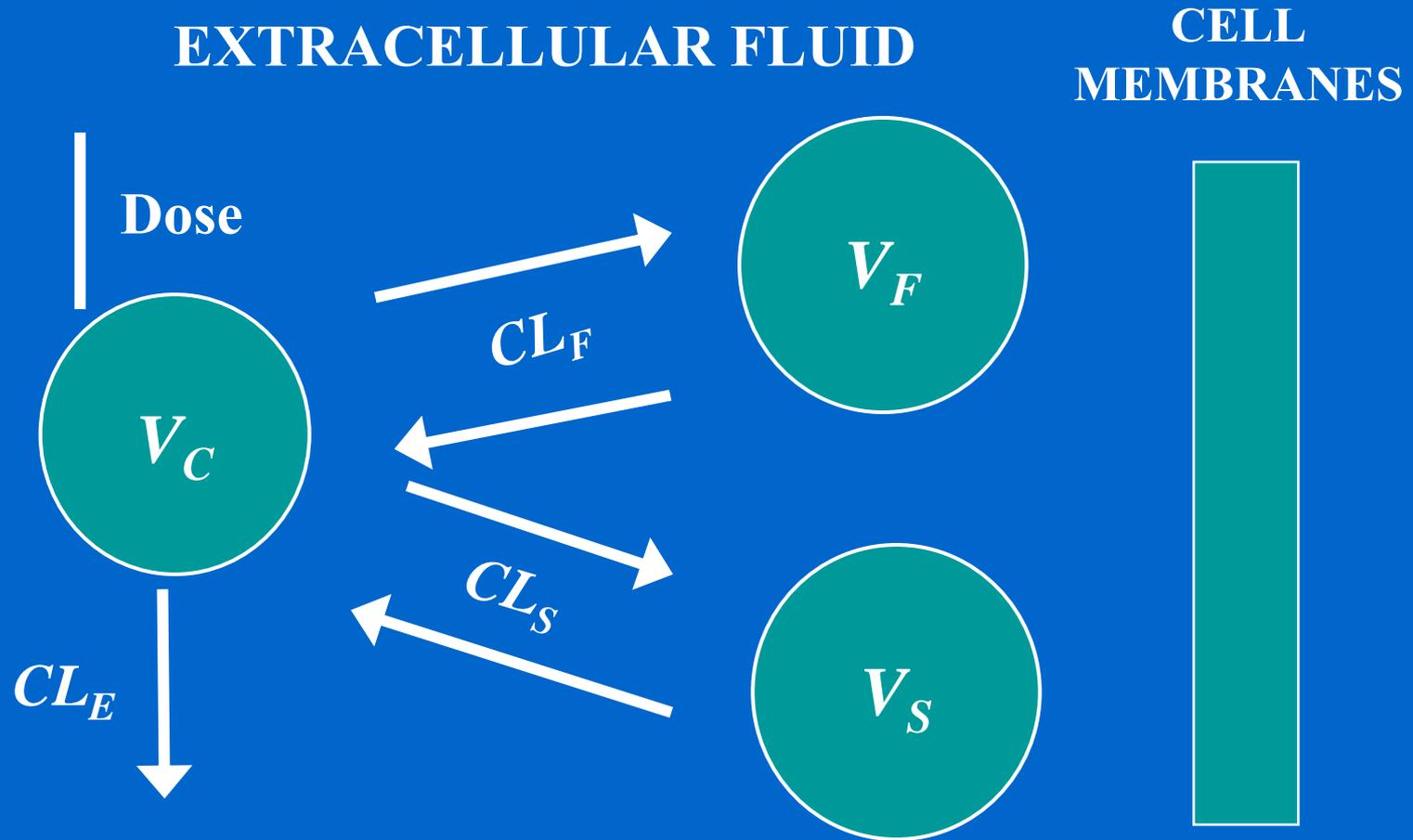
Hct = hematocrit

Analysis of **Inulin** Kinetics with a 2-Compartment Model*



* Gaudino M. Proc Soc Exper Biol Med 1949;70:672-4.

3-Compartment Model of Inulin Kinetics



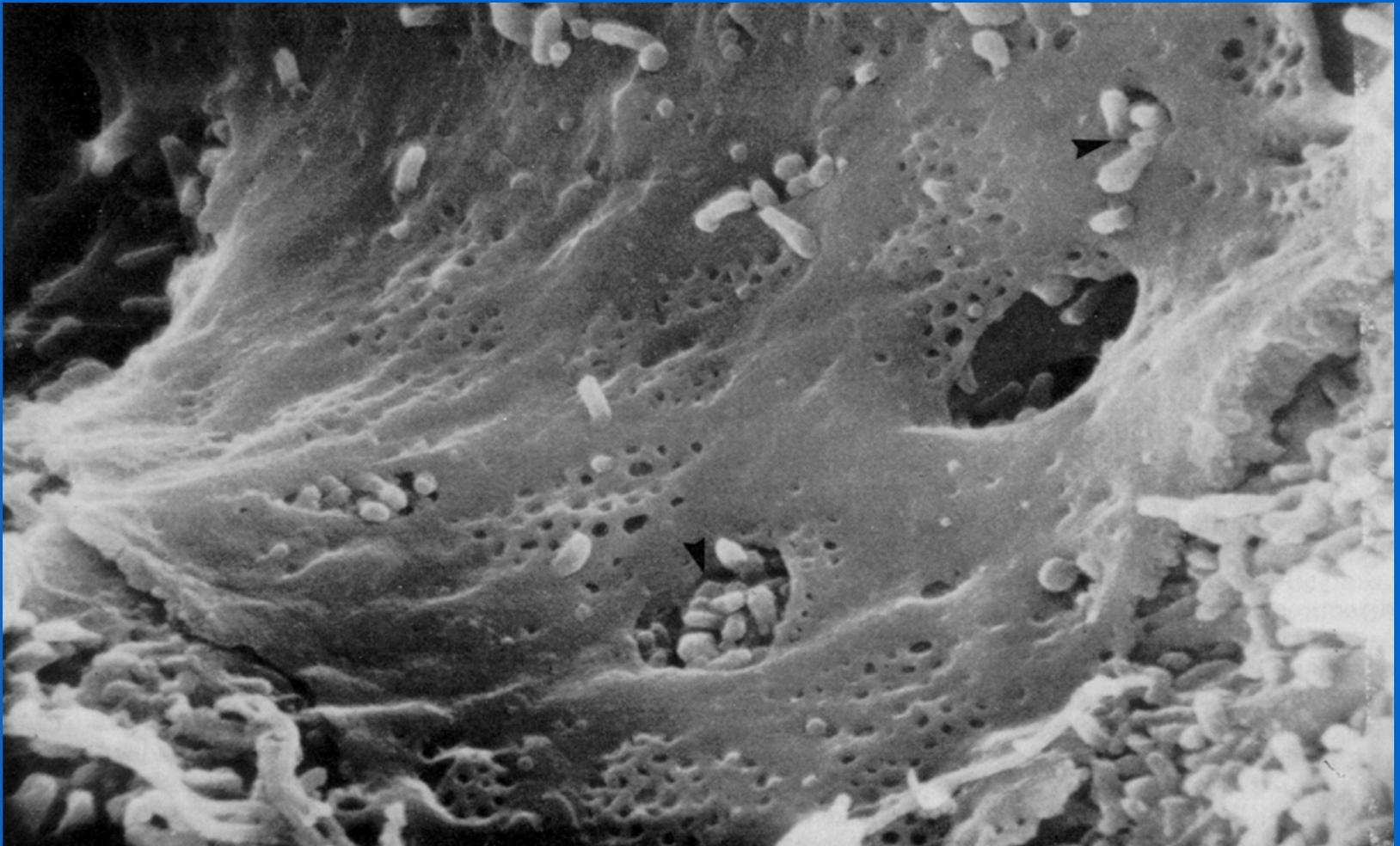
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Basis for **Kinetic Heterogeneity** of Interstitial Fluid Space

EFFECTIVE PORE SIZE	CAPILLARY STRUCTURE	PRIMARY LOCATION
LARGE	FENESTRATED	SPLANCHNIC BED
SMALL	CONTINUOUS	SOMATIC TISSUES

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ENDOTHELIAL FENESTRAE IN HEPATIC SINUSOIDS

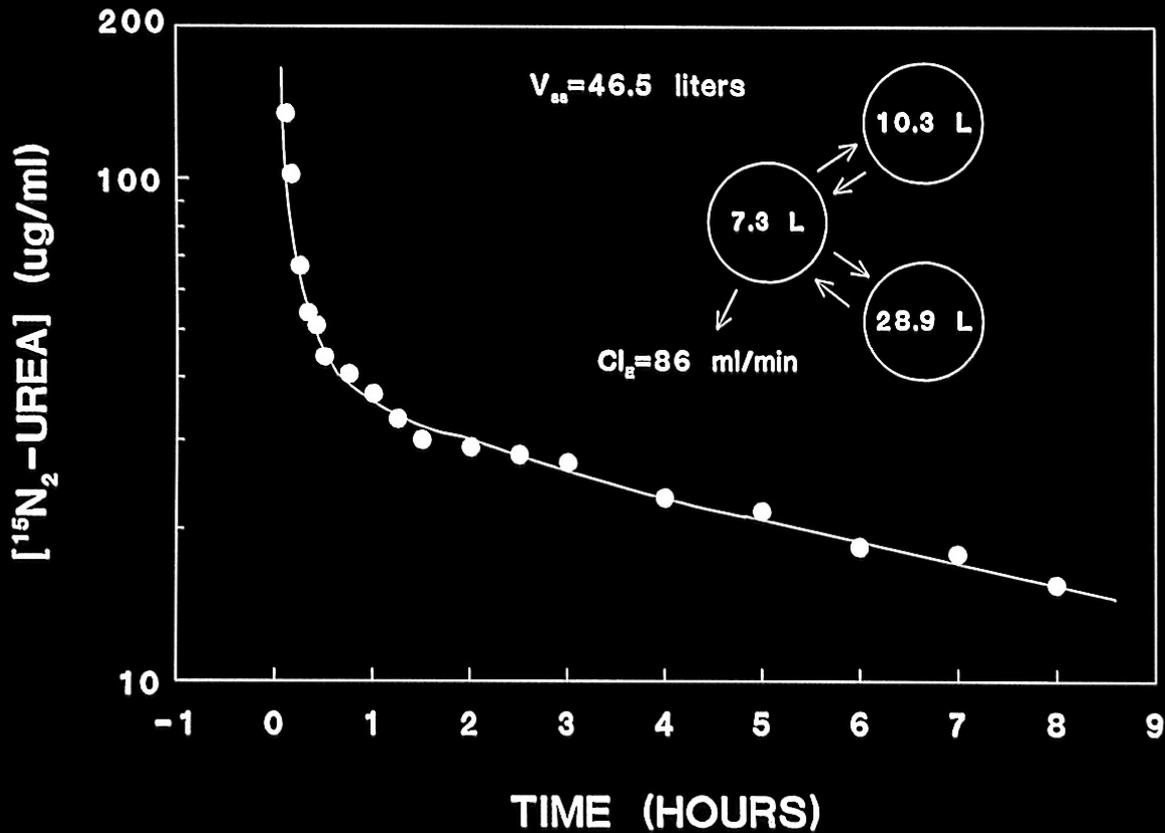


INTERENDOTHELIAL CELL JUNCTION IN CONTINUOUS CAPILLARY

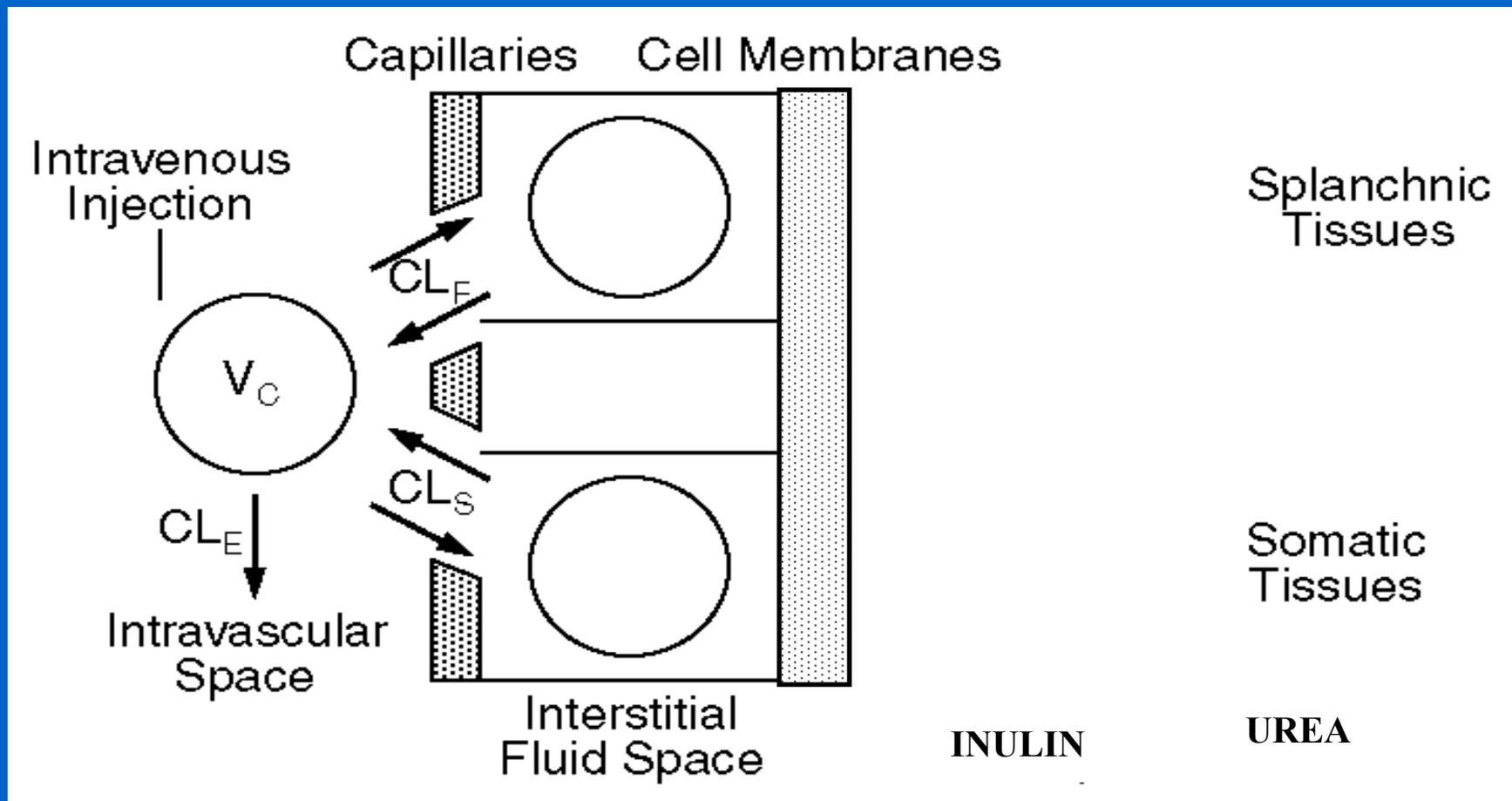


UREA-¹⁵N₂ KINETICS IN A NORMAL SUBJECT

UREA-¹⁵N₂ KINETICS



Multicompartment Model of Inulin and Urea Kinetics*



* From Atkinson AJ Jr, et al. Trends Pharmacol Sci 1991;12:96-101.

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ROLE OF *TRANSCAPILLARY EXCHANGE*

The **central** compartment for both **urea** and **inulin** is the **intravascular** space.

Therefore, **transcapillary exchange** is the **rate-limiting** step in the distribution of urea and inulin to the **peripheral** compartments of the mammillary **3-compartment model**.

RENKIN EQUATION*

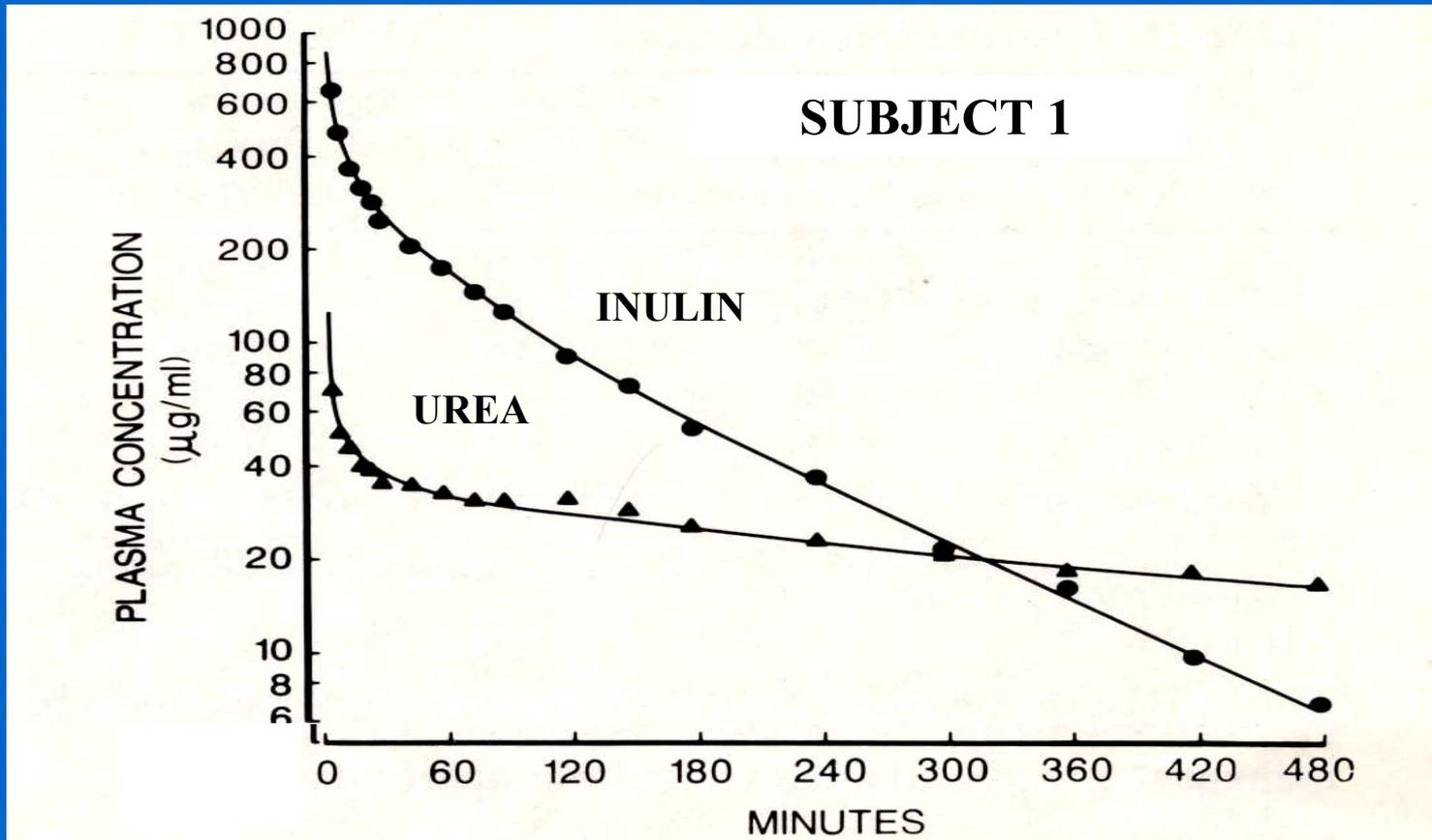
$$Cl = Q (1 - e^{-P/Q})$$

Q = capillary blood flow

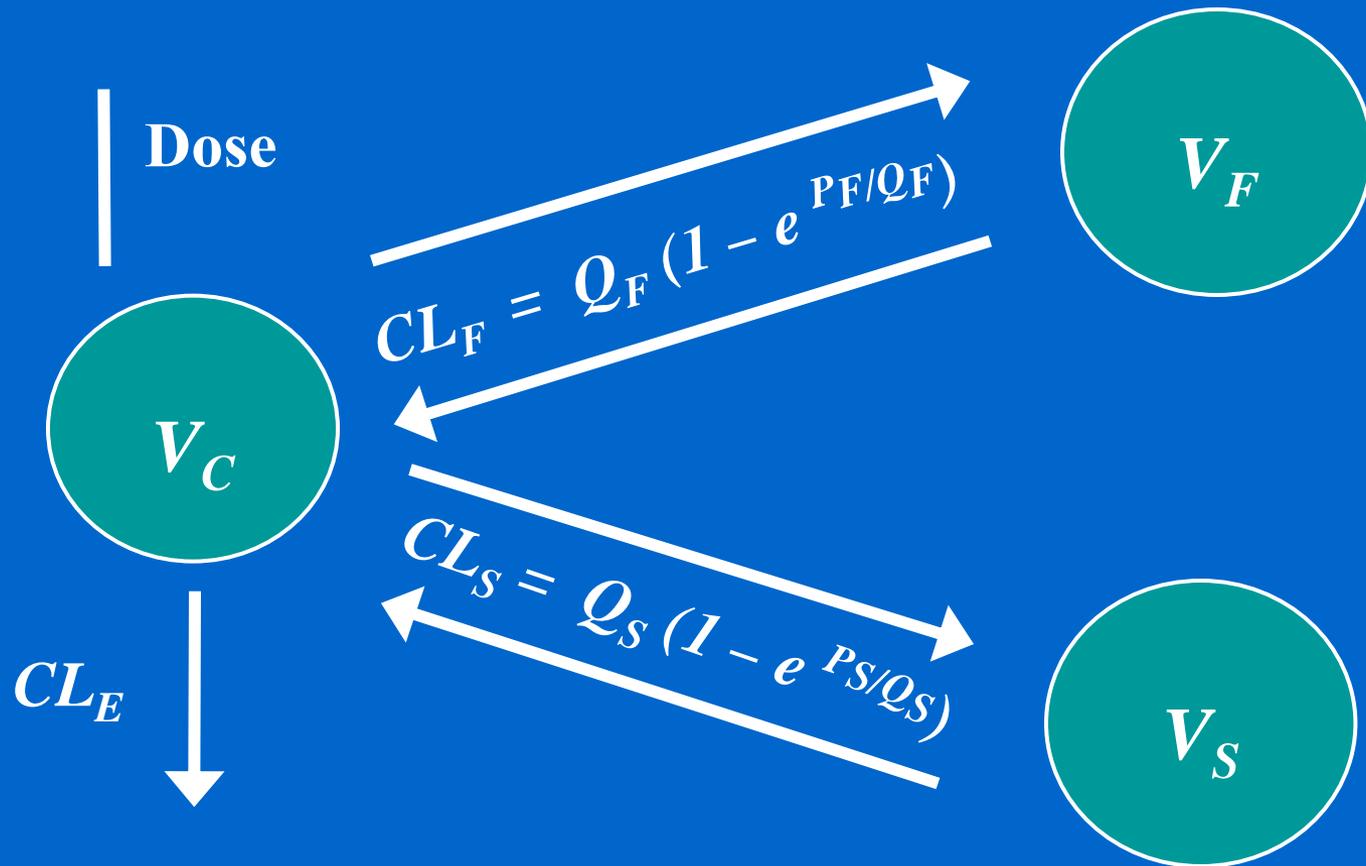
P = capillary permeability coefficient-surface area product (sometimes denoted P•S).

* From Renkin EM. Am J Physiol 1953;183:125-36.

SIMULTANEOUS ANALYSIS OF INULIN AND UREA-¹⁵N₂ KINETICS



3-COMPARTMENT MODEL



For Each Peripheral Compartment

3 UNKNOWNNS:

$$Q, P_U, P_I$$

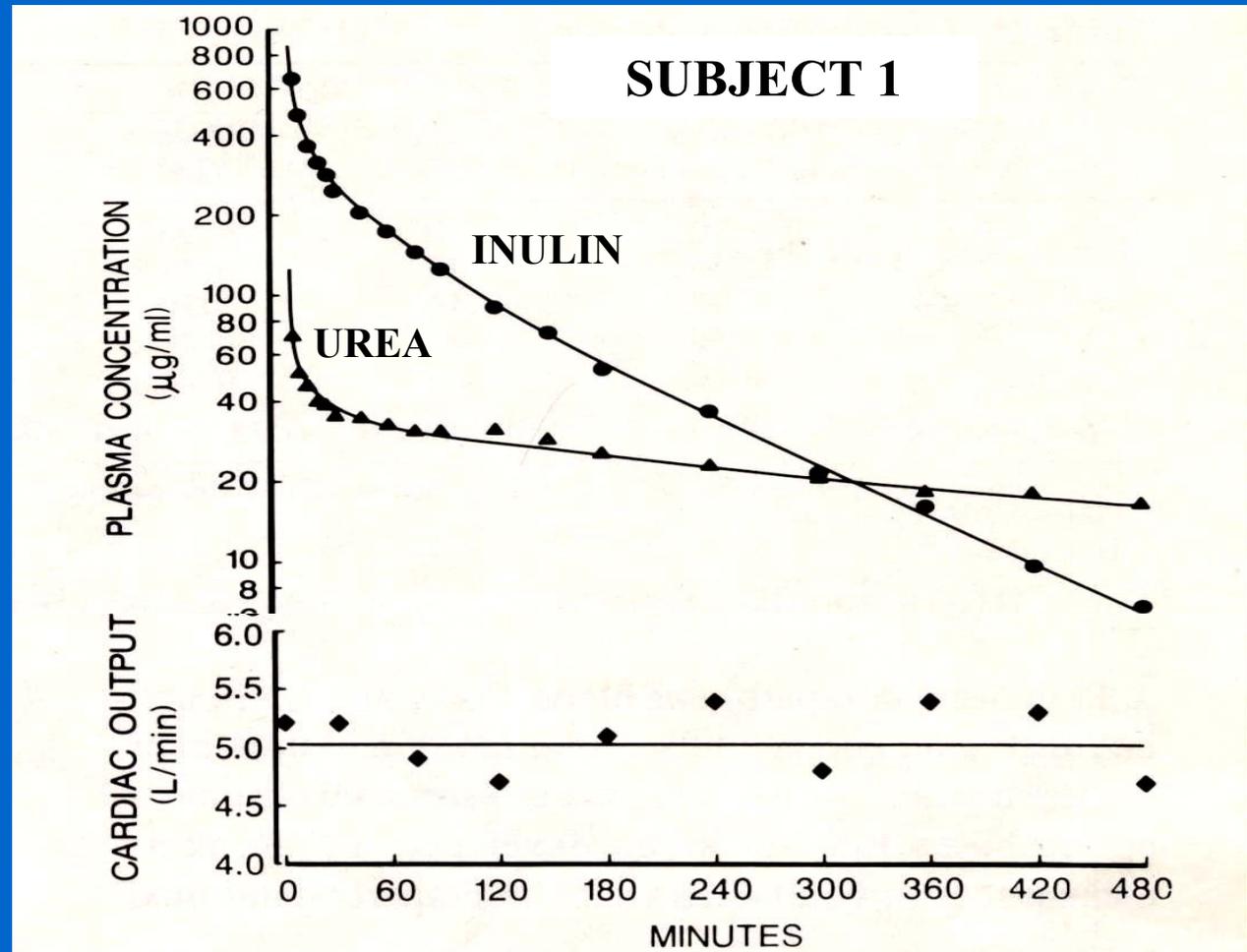
3 EQUATIONS:

$$P_U = Q \ln \left[\frac{Q}{Q - CI_U} \right]$$
$$P_I = Q \ln \left[\frac{Q}{Q - CI_I} \right]$$
$$P_U / P_I = D_U / D_I$$

U = urea; I = inulin

D = free water diffusion coefficient

SIMULTANEOUS ANALYSIS OF INULIN AND UREA-¹⁵N₂ KINETICS



How does
 $Q_F + Q_S$
compare
with C.O.?

CARDIAC OUTPUT AND COMPARTMENTAL BLOOD FLOWS*

	Q_F	Q_S	$Q_F + Q_S$	
	L/min	L/min	L/min	% CO
MEAN[†]	3.87	1.52	5.39	99

† MEAN OF 5 SUBJECTS

*** From Odeh YK, et al. Clin Pharmacol Ther 1993;53;419-25.**

TRANSCAPILLARY EXCHANGE Mechanisms

TRANSFER OF SMALL MOLECULES (M.W. < 6,000 Da):

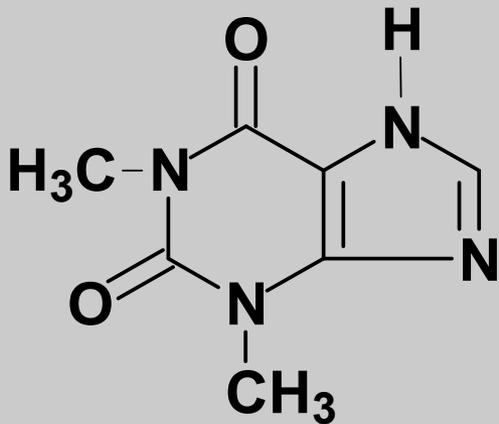
- **Transfer proportional to D**
 - Polar, uncharged (urea, inulin)
- **Transfer rate < predicted from D**
 - Highly charged (quaternary compounds)
 - Interact with pores (procainamide)
- **Transfer rate > predicted from D**
 - Lipid soluble compounds (anesthetic gases)
 - Facilitated diffusion (theophylline)

Urea and Theophylline Diffusion Coefficients*

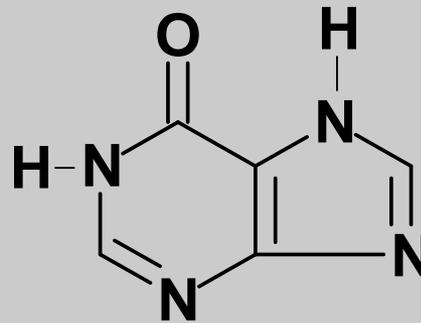
	MOLECULAR WEIGHT (DALTONS)	CORRECTED STOKES- EINSTEIN RADIUS (Å)	D_m @ 37° C (x 10⁻⁵ cm²/sec)
UREA	60	2.2	1.836
THEOPHYLLINE	180	3.4	1.098

* From Belknap SM, et al. J Pharmacol Exp Ther 1987;243;963-9.

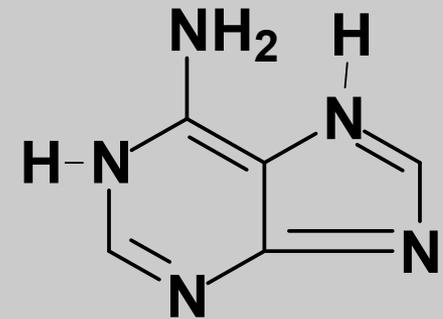
PRESUMED *CARRIER-MEDIATED* TRANSCAPILLARY EXCHANGE



THEOPHYLLINE



HYPOXANTHINE



ADENINE

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GOALS OF DRUG DISTRIBUTION LECTURE

- **Significance of drug distribution volumes**
- **Physiologic basis of multi-compartment pharmacokinetic models**
- **Clinical implications of drug distribution kinetics**

SIGNIFICANCE OF DRUG DISTRIBUTION *RATE*

1. Affects toxicity of IV injected drugs

Theophylline, lidocaine

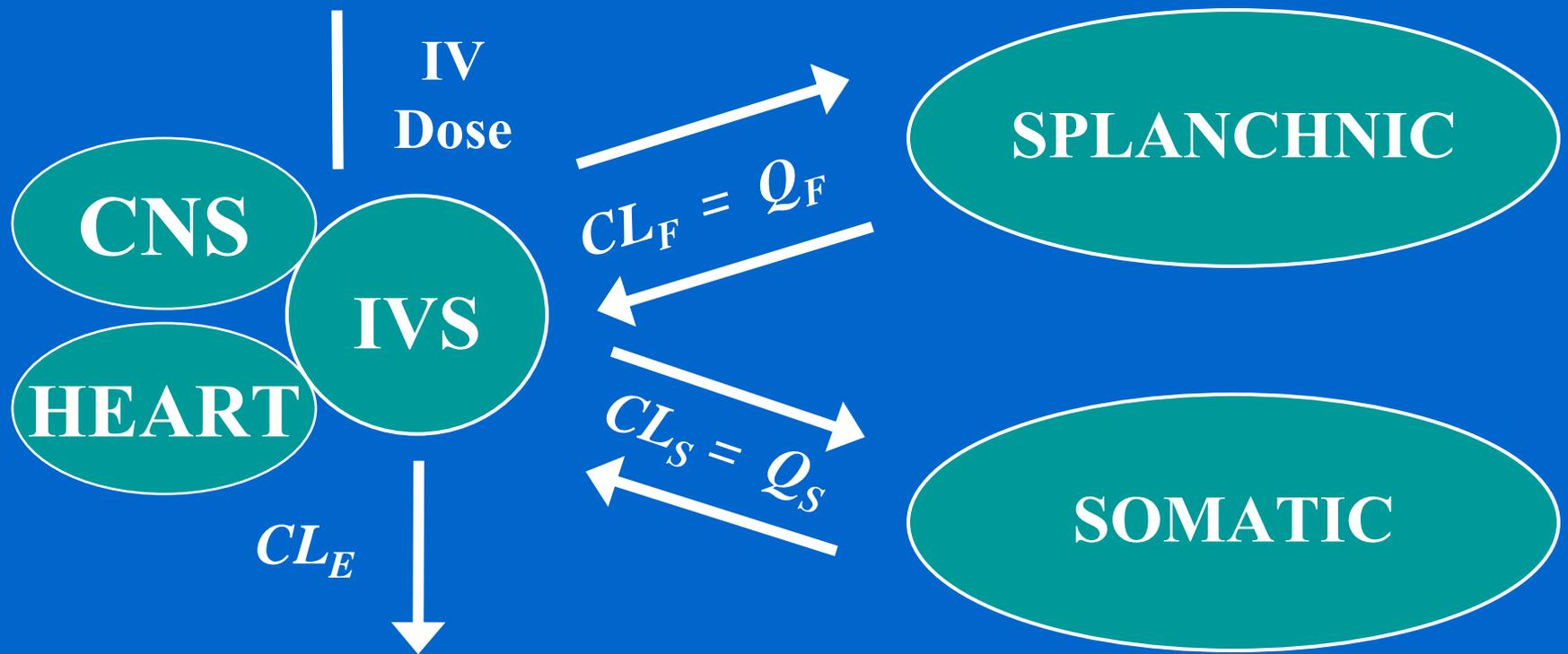
2. Delays onset of drug action

Insulin, digoxin

3. Terminates action after IV bolus dose

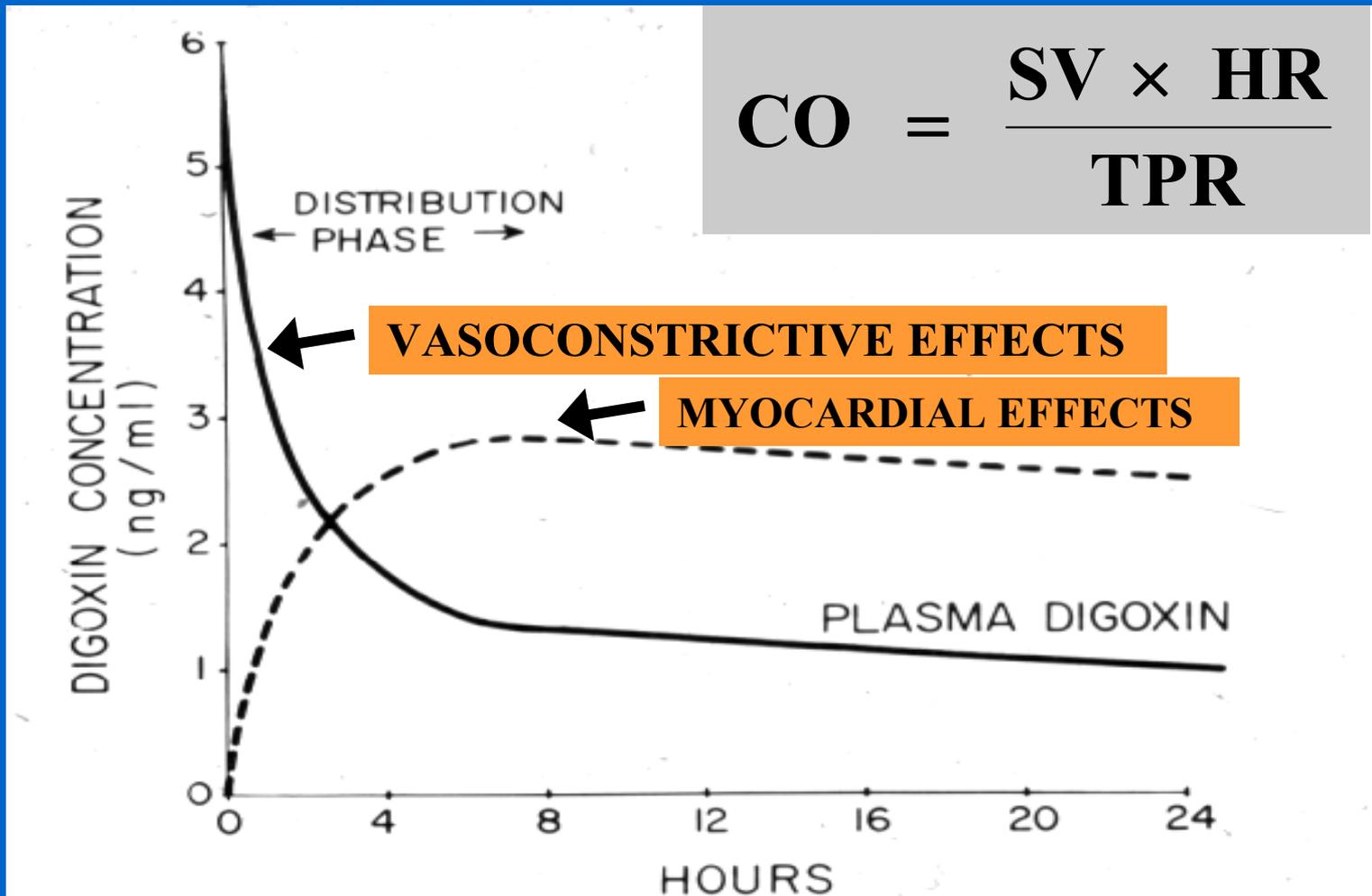
Thiopental, lidocaine

PK Model of **THEOPHYLLINE** Distribution

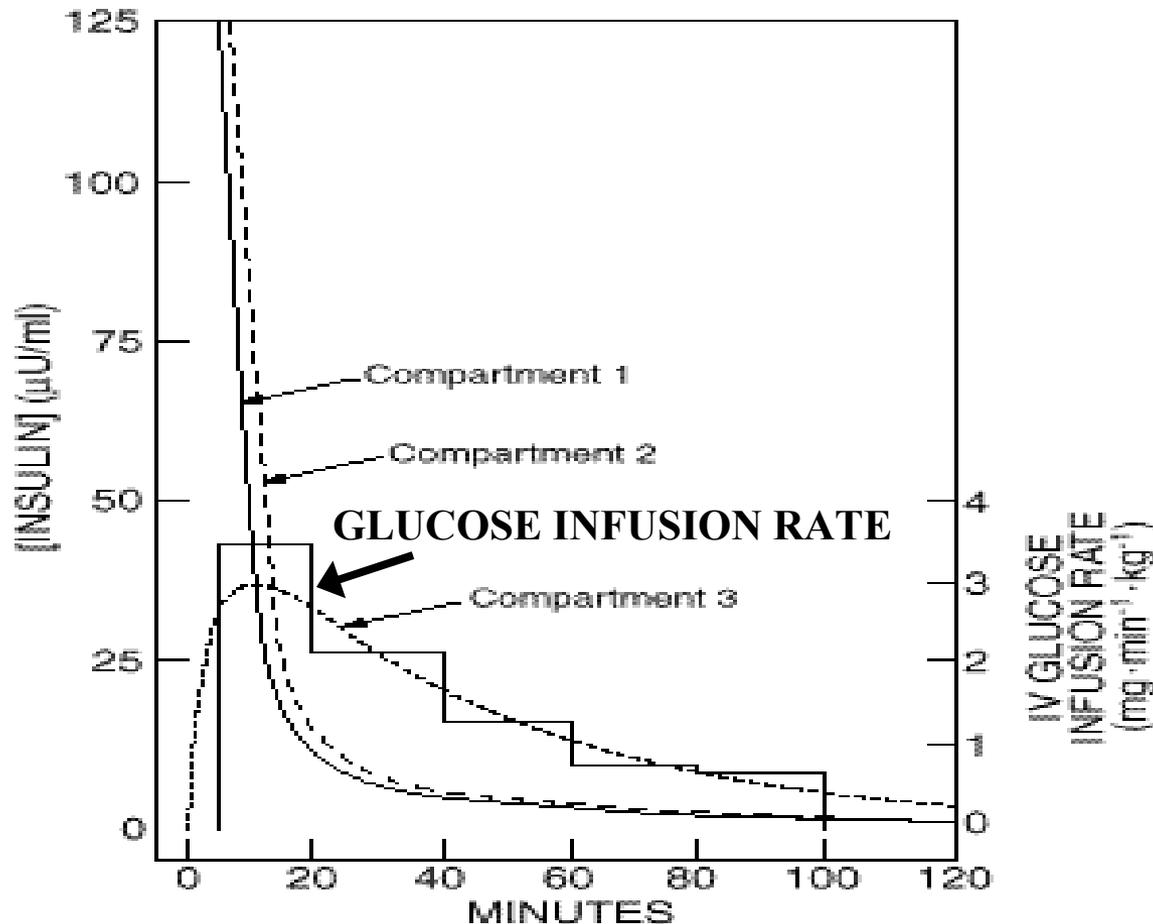


$$CO = Q_F + Q_S$$

DIGOXIN is NOT the First Drug Given to Patients with Acute Pulmonary Edema

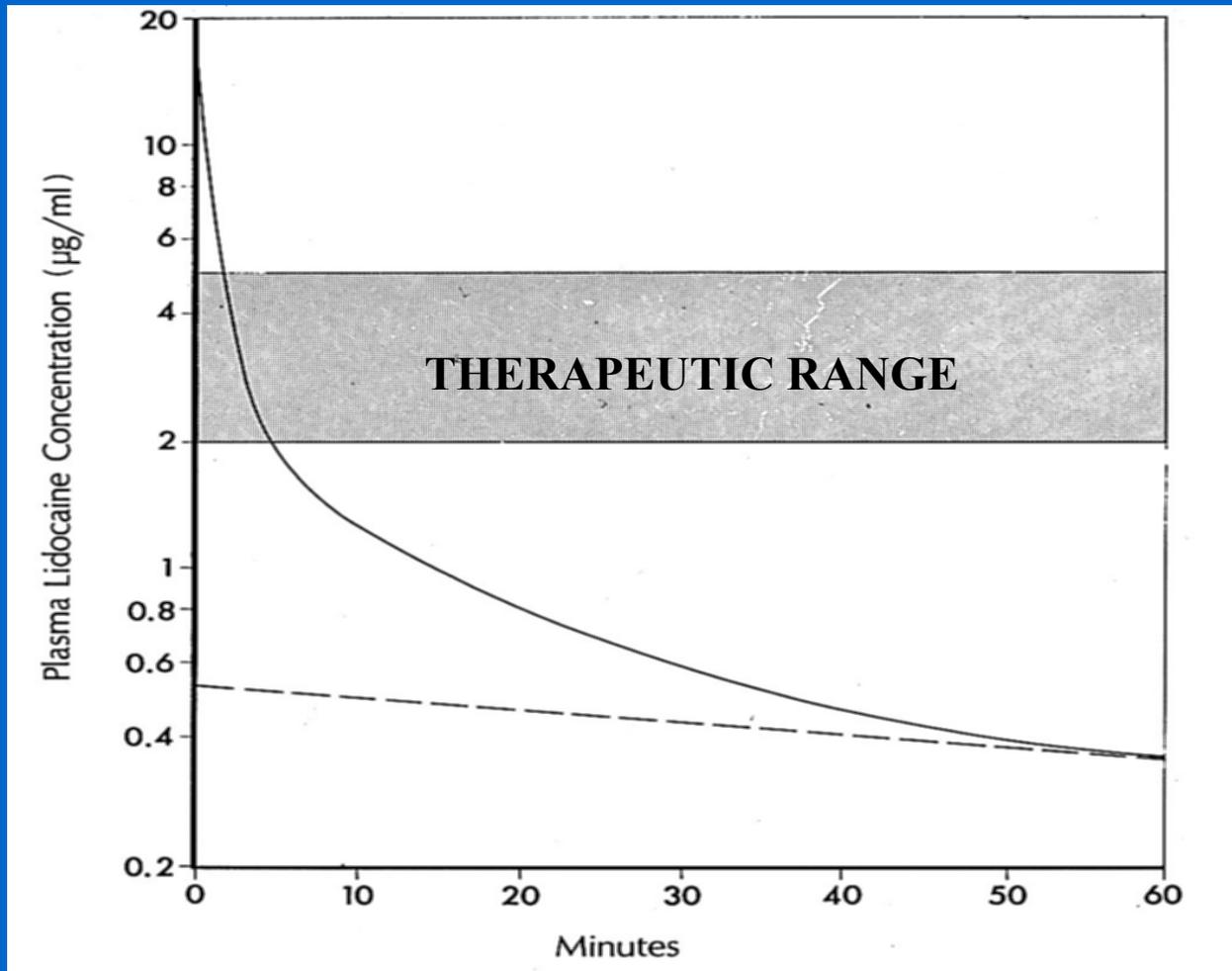


PK-PD Study of **INSULIN** Enhancement of Skeletal Muscle **Glucose Uptake***



* From Sherwin RS, et al. J Clin Invest 1974;53:1481-92.

DISTRIBUTION TERMINATES EFFECT BOLUS LIDOCAINE DOSE*



* From Atkinson AJ Jr. In: Melmon KL, ed. Drug Therapeutics: Concepts for Physicians, 1981:17-33.

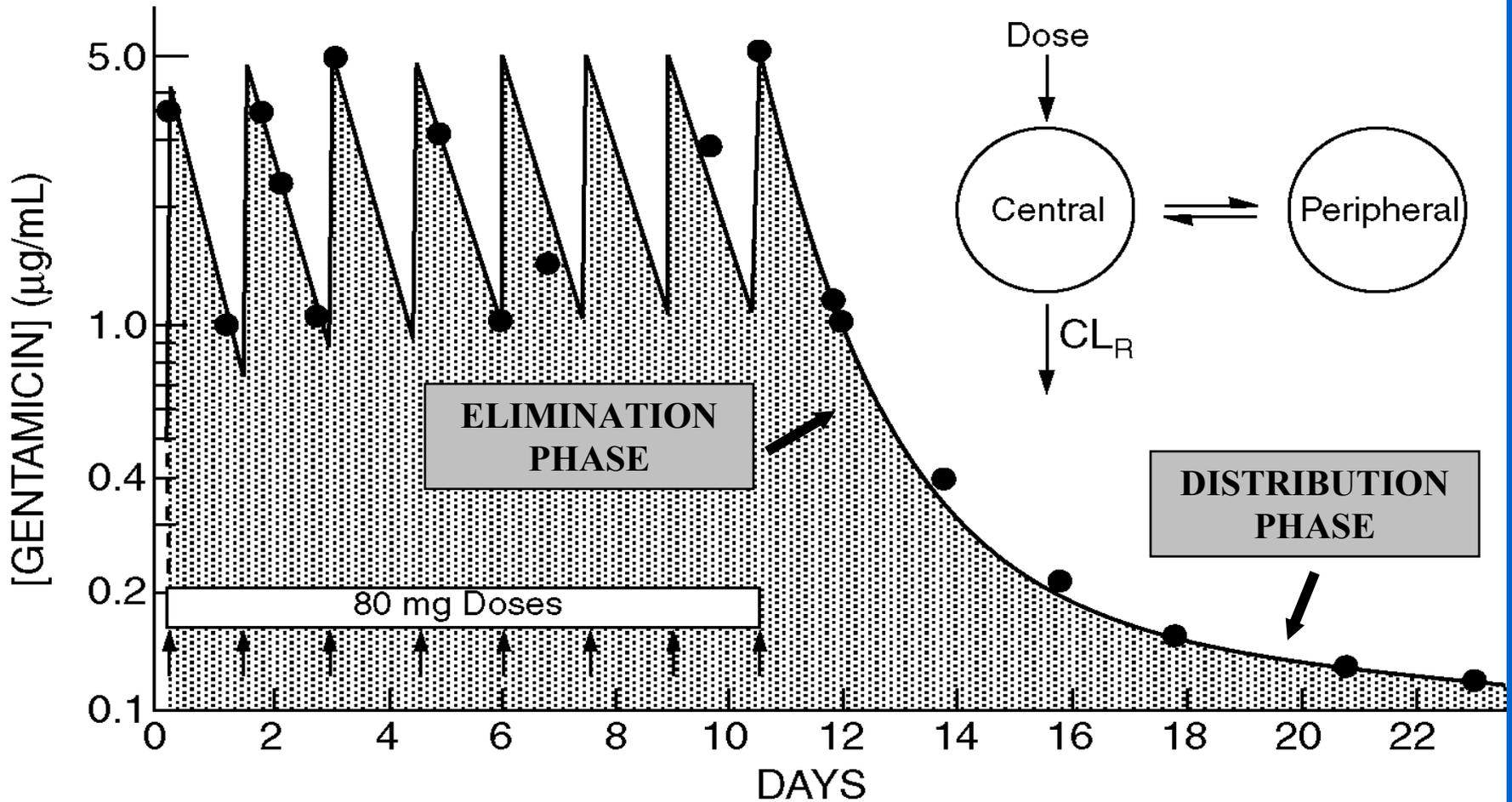
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CONSEQUENCES OF *VERY SLOW* DRUG DISTRIBUTION

- **“Flip-Flop” Kinetics**
- **Effective Half-Life**
- **Pseudo Dose Dependency**

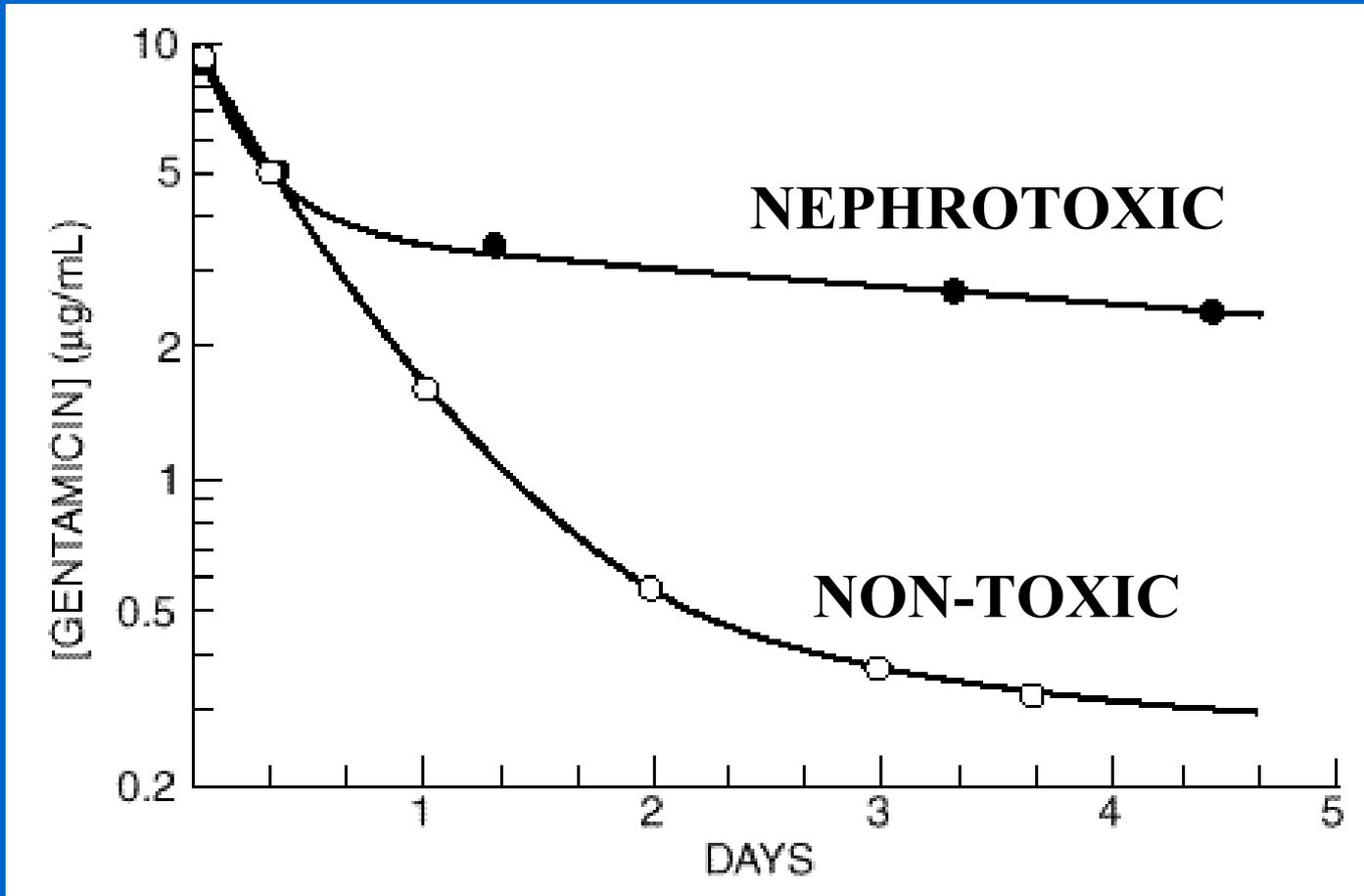
GENTAMICIN

Elimination Phase Precedes Distribution Phase*



* From Schentag JJ, et al. JAMA 1977;238:327-9.

GENTAMICIN ELIMINATION Nephrotoxic vs. Non-Toxic Patient*



* From Coburn WA, et al. J Pharmacokinet Biopharm 1978;6:179-86.

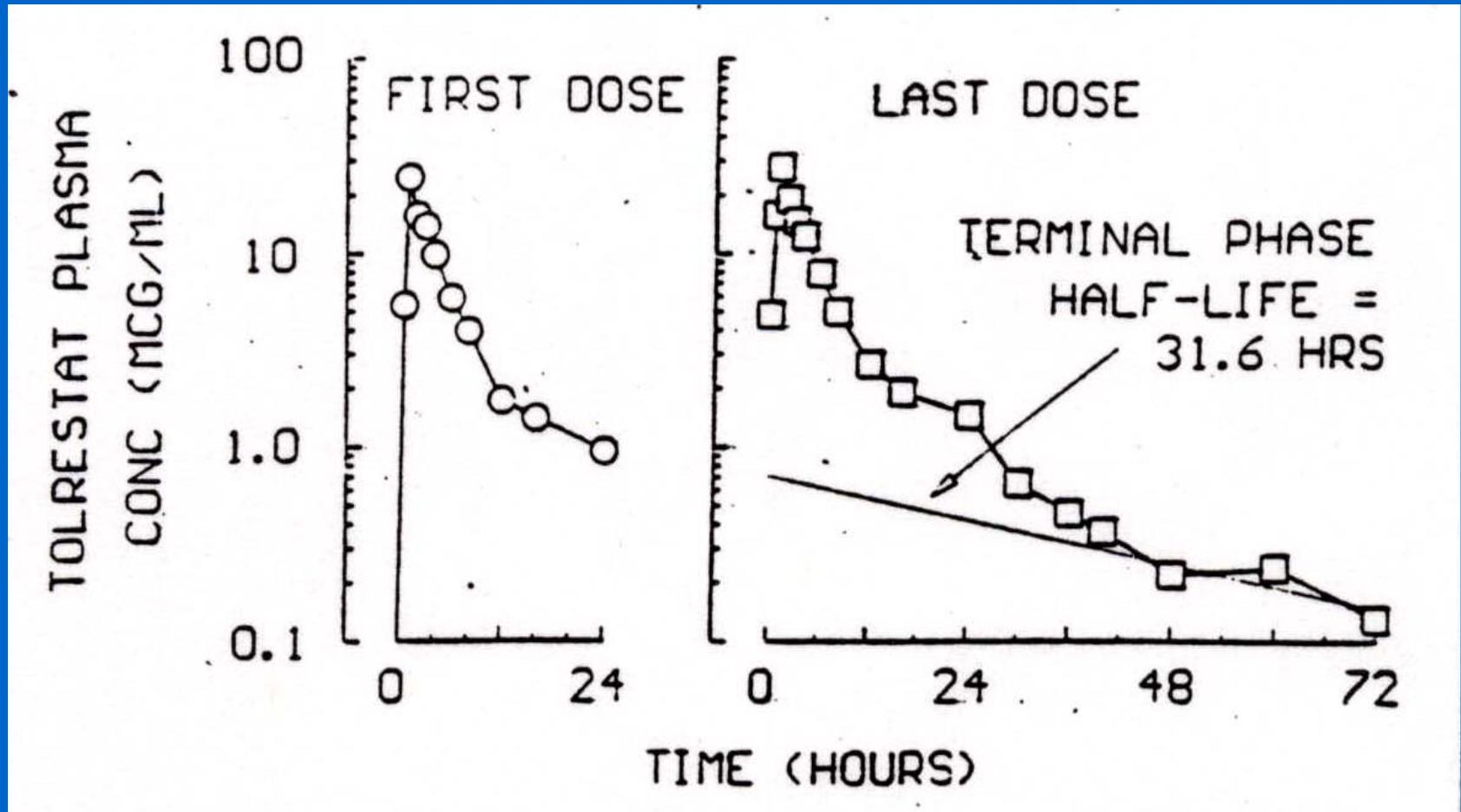
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CONSEQUENCES OF *VERY SLOW* DRUG DISTRIBUTION

- “Flip-Flop” Kinetics
- **Effective Half-Life**
- Pseudo Dose Dependency

TOLRESTAT

Cumulation with Repeated Dosing*



*From Boxenbaum H, Battle M: J Clin Pharmacol 1995;35:763-6.

CUMULATION FACTOR

$$CF = \frac{1}{\left(1 - e^{-k\tau}\right)}$$

TOLRESTAT CUMULATION

Predicted C.F. from $T_{1/2} = 31.6$ hr: 4.32

Observed C.F.: 1.29

EFFECTIVE HALF- LIFE*

$$k_{\text{eff}} = \frac{1}{\tau} \ln \left(\frac{\text{CF}_{\text{obs}}}{\text{CF}_{\text{obs}} - 1} \right)$$

$$t_{1/2\text{eff}} = \frac{\ln 2}{k_{\text{eff}}}$$

* From Boxenbaum H, Battle M. J Clin Pharmacol 1995;35:763-66.

EFFECTIVE HALF-LIFE OF TOLRESTAT*

Since $\tau = 12$ hr and Observed CF = 1.29:

$$k_{\text{eff}} = \frac{1}{12} \ln\left(\frac{1.29}{1.29-1}\right) = 0.124 \text{ hr}^{-1}$$

$$t_{1/2\text{eff}} = \frac{\ln 2}{0.124} = 5.6 \text{ hr}$$

* From Boxenbaum H, Battle M. J Clin Pharmacol 1995;35:763-66.

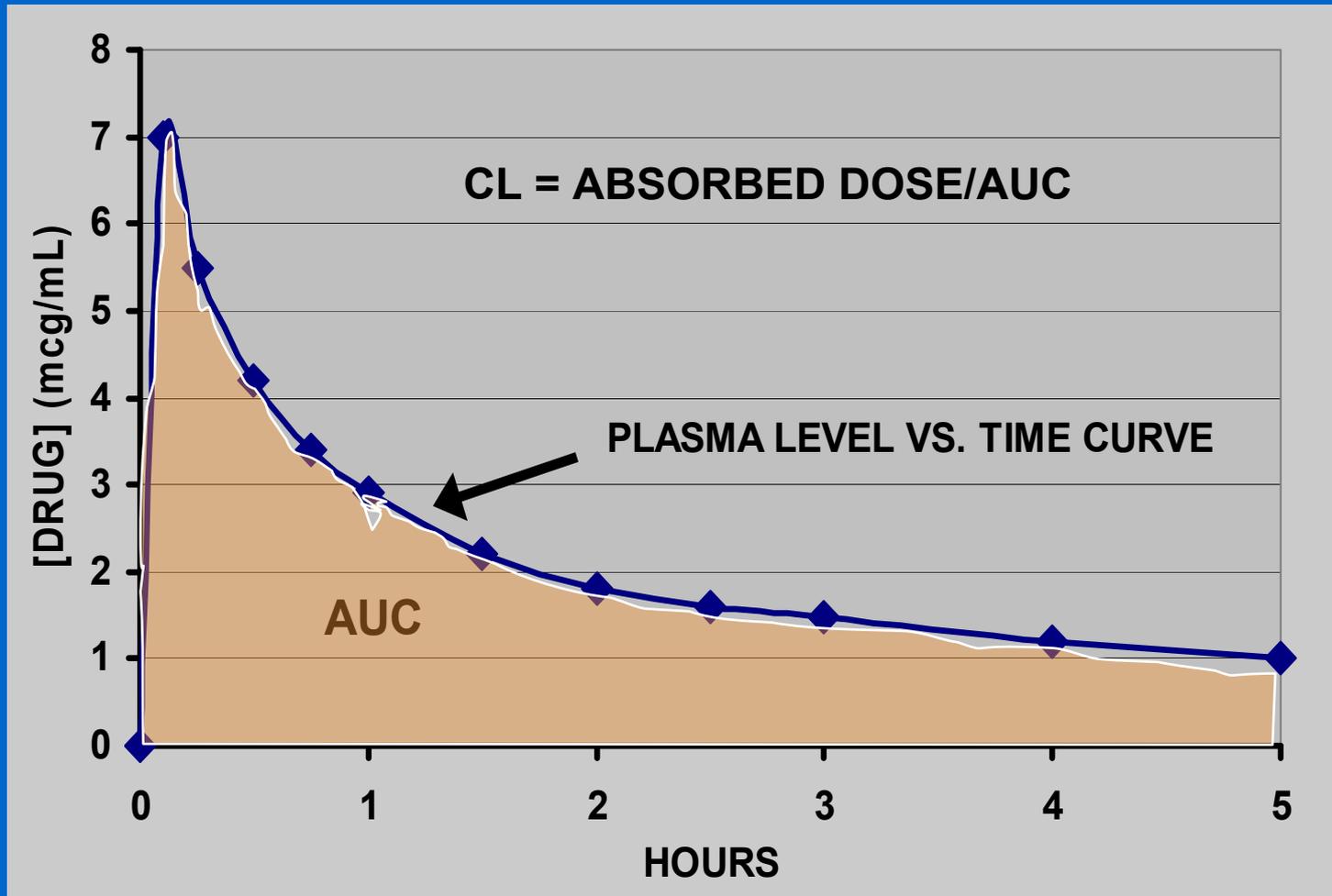
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CONSEQUENCES OF *VERY SLOW* DRUG DISTRIBUTION

- “Flip-Flop” Kinetics
- Effective Half-Life
- Pseudo Dose Dependency

AREA UNDER THE CURVE

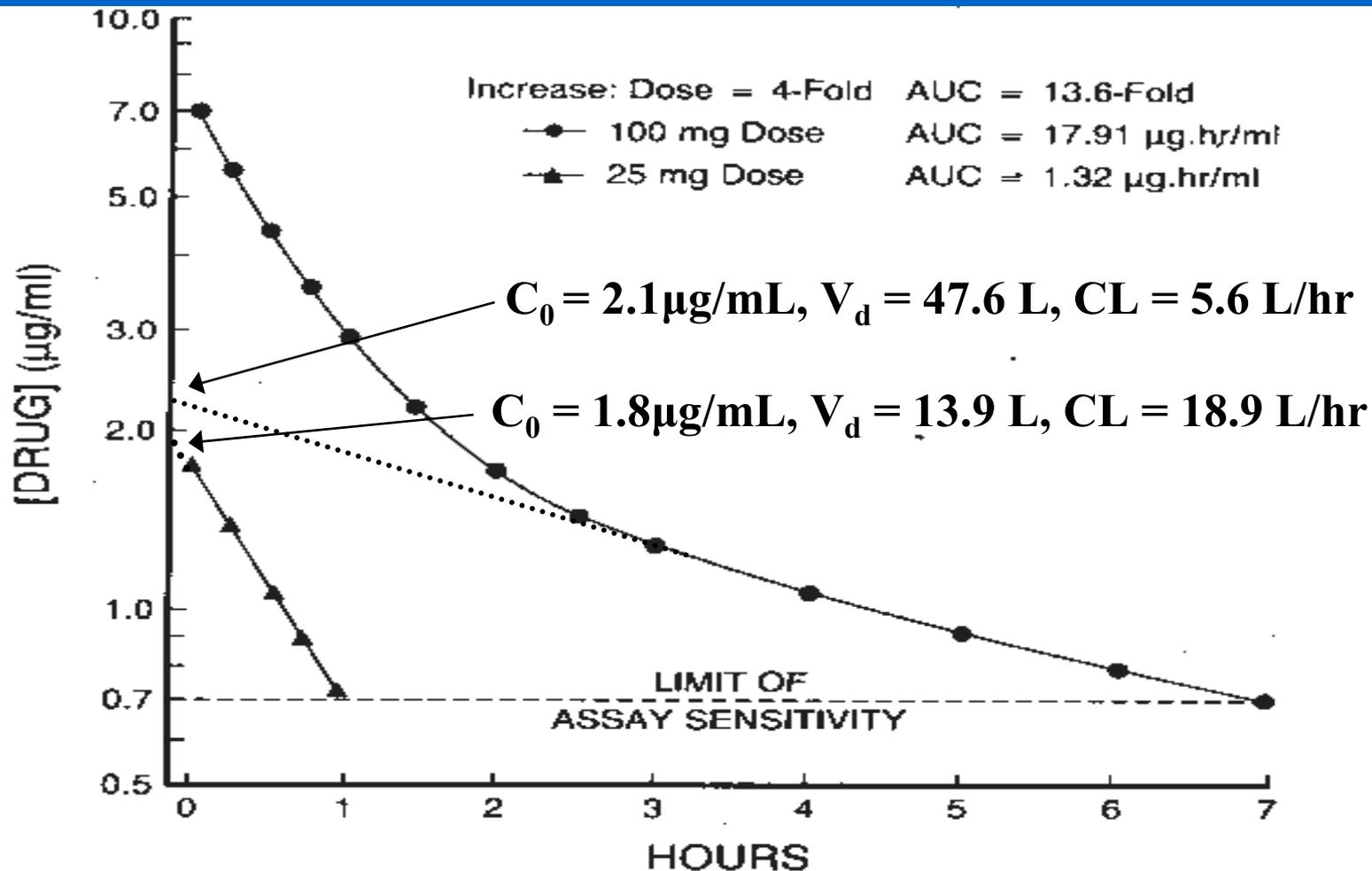
Measure of Dose Proportionality



HYPOTHETICAL Phase I Trial Results

	DOSE 1	DOSE 2	INCREASE
DOSE (mg)	25	100	4 x ↑
AUC ($\mu\text{g}\cdot\text{hr}/\text{mL}$)	1.32	17.91	13.6 x ↑

Dependency of PK Estimates on Identified Terminal Phase



DISTRIBUTION VOLUME

Representative Macromolecules

MACROMOLECULE	MW (kDa)	V₁ (mL/kg)	V_{d(ss)} (mL/kg)
INULIN	5.2	55	164
FACTOR IX (FIX)	57	136	271
INTERLEUKIN-2 (IL-2)	15.5	60	112
INTERLEUKIN-12 (IL-12)	53	52	59
GRANULOCYTE COLONY STIMULATING FACTOR (G-CSF)	20	44	60
RECOMBINANT TISSUE PLASMINOGEN ACTIVATOR (RT-PA)	65	59	106

CLOTTING FACTOR PHARMACOKINETICS*

- “The $V_{d(ss)}$ always **exceeds** the actual **plasma volume**, implying that **no drug**, not even large molecular complexes as F-VIII, is **entirely confined to the plasma space.**”
- “A too **short blood sampling** protocol gives **flawed results** not only for terminal $T_{1/2}$ but also for the model independent parameters.”

* Berntorp E, Björkman S. Haemophilia 2003;9:353-9.